

# **Technological Change, Household Debt, and Distribution**

**Eric Kemp-Benedict and Yun K. Kim**

---

**WORKING PAPER 2018-02**

---

**DEPARTMENT OF ECONOMICS**

**UNIVERSITY OF MASSACHUSETTS BOSTON**

---



# Technological Change, Household Debt, and Distribution\*

Eric Kemp-Benedict<sup>1</sup> and Yun K. Kim<sup>2</sup>

<sup>1</sup>Stockholm Environment Institute, Somerville, MA, US

<sup>2</sup>University of Massachusetts, Boston, MA, US

## Abstract

We present a stylized model to explore the interaction between household debt, the functional income distribution, and technological change. We assume that weak labor bargaining power allows firms to set their markups in order to meet a target profit rate. At a low wage share, workers' households are assumed to have limited flexibility in meeting financial goals, so household indebtedness tends to rise as the wage share falls. Rising indebtedness further lowers labor's bargaining power, a phenomenon that was observed in the wave of financialization that began in the late 20th century. Thus, rising debt levels allow firms even greater freedom to raise their target profit rate. We find that the dynamics can be either stable or unstable, with the potential for a self-reinforcing pattern of rising household indebtedness and falling wage share, consistent with trends in the US from the 1980s onward. The unstable cycle can be triggered by increased willingness by workers to incur debt and rising influence of household indebtedness on labor's bargaining strength and income distribution. The model can shed some light on widely-observed trends over recent decades regarding household indebtedness, inequality, and technological changes in the US, and potentially in other OECD countries.

## 1 Introduction

The US economy at the end of the 20th century was characterized by a rising profit share (and falling wage share) and growing levels of household debt as

---

\*Earlier versions of this paper were presented at the Annual Meetings of Eastern Economic Associations, March 1-4, 2018. We would like to thank conference participants for their helpful comments. We also like to thank Daniele Tavani for very helpful comments to improve the paper.

part of a broader process of financialization (shown in figure 2, below) (Foster, 2007; Hein, 2013). Multiple channels connect the debt level and income of workers' households to changes in their indebtedness. Different channels may be active under different conditions, depending on workers' principal motivation to borrow, whether consumption goals or financial considerations.

The latter part of the 20th century was also characterized by slowing labor productivity growth, starting in the 1970s as shown in figure 1. Annual average non-farm labor productivity growth for the US from 1948 to 2016 exhibits a single structural break in 1973<sup>1</sup>. Fitting a constant model, average growth fell from an average of 2.8% per year between 1948 and 1973 to 1.7% per year between 1974 and 2016.

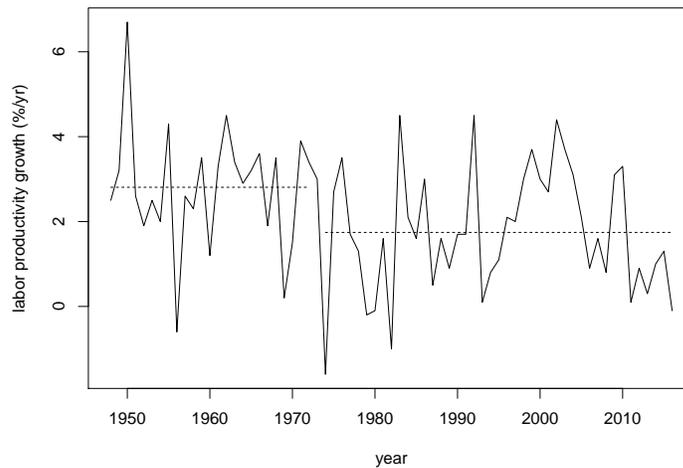


Figure 1: Labor Productivity Growth

Figure 2 presents data for the wage share calculated from the Penn World Tables (PWT) (Feenstra et al., 2013) and the household debt-to-capital ratio

<sup>1</sup>Using US Bureau of Labor Statistics series PRS85006092. The break was detected using the R package “strchange”, which automatically detects the timing of structural breaks in linear regression models (Zeileis et al., 2003).

estimated from data drawn from FRED and PWT, while figure 3 shows the trend in capital productivity and the profit rate computed from PWT data.<sup>2</sup> As seen in the figures, the wage share began to fall after around 1970, stabilized from the mid-1980s to the late 1990s, rose briefly during the dot-com bubble and then began to fall again after 2000. Household debt has generally risen in the postwar period, but more slowly from the mid-1960s to the early 1980s than in other periods. The functional income distribution was comparatively steady until the mid-1970s, so the profit rate in figure 3 generally kept pace with changes in capital productivity. While both began to fall around 1966, in the mid-1970s, the profit rate began to rise relative to capital productivity, as the wage share declined.

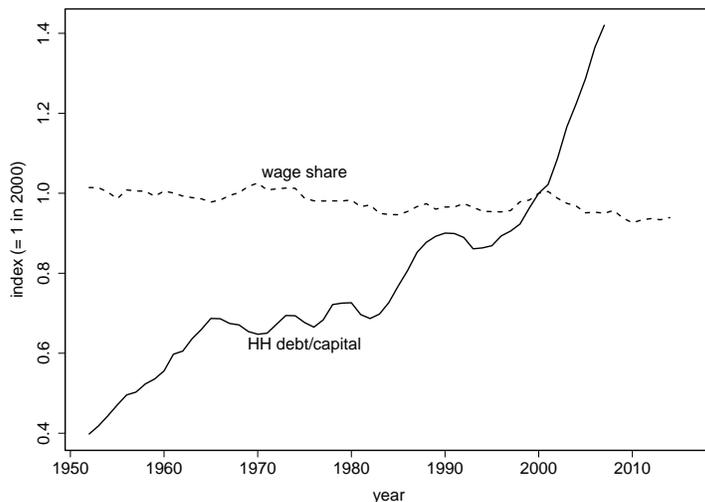


Figure 2: Wage share and household debt-to-capital stock ratio

<sup>2</sup>We show the debt-to-capital ratio in order to be consistent with the theoretical model developed in this paper, but we observe a very similar trend for the more conventional debt-income ratio. Capital productivity is calculated as GDP divided by the capital stock. The profit rate is calculated as  $(1 - \text{wage share})$  multiplied by capital productivity. The debt-to-capital stock ratio  $d_W$  is calculated as capital productivity multiplied by the ratio of household debt (mortgage debt plus consumer debt from the Federal Reserve) to GDP (from the National Income and Product Accounts NIPA).

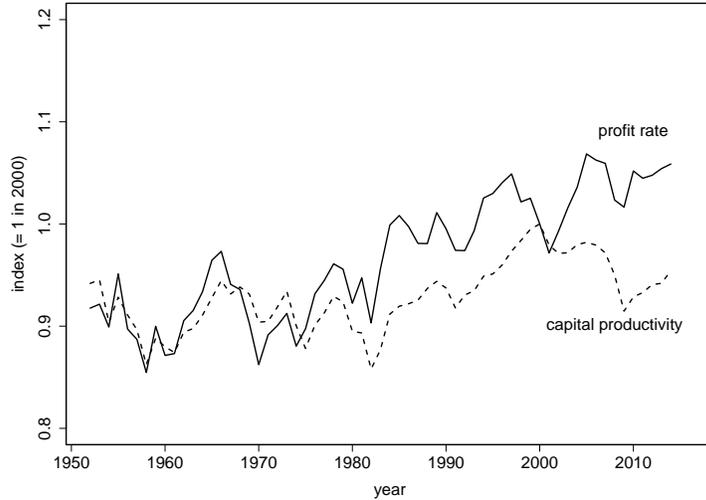


Figure 3: Capital productivity and profit rate

Cost-share induced theories of technological change (Kemp-Benedict, in press) can connect these observations, as a lower labor share can induce a slower rate of labor productivity change. However, as noted above, capital productivity had begun to decline even earlier, around 1966. While the observed trends can be explained by different theories, the temporal sequence suggests that the fall in capital productivity was the trigger. Comparatively strong unions in the 1960s prevented firms from maintaining profit rates, but high unemployment in the 1970s in the wake of oil crisis weakened bargaining power, allowing firms to increase profits at the expense of wages. By the early 1980s, the profit rate began to increase. At around the same time, household debt began to rise as well (Kim, 2013).

In this paper we present a stylized model to explore the interaction between household debt, the functional income distribution, and productivity. We assume that weak labor bargaining power allows firms to set their markups in order to meet a target profit rate. At a low wage share, workers' households

are assumed to have limited flexibility in meeting financial goals, so household indebtedness tends to rise as the wage share falls. Rising indebtedness further lowers labor's bargaining power (Bonefeld, 1995, p.69), a phenomenon that was observed in the wave of financialization that began in the late 20th century (Slater and Spencer, 2014, p.142). Thus, rising debt levels allow firms even greater freedom to raise their target profit rate. These connections are shown in Figure 4.

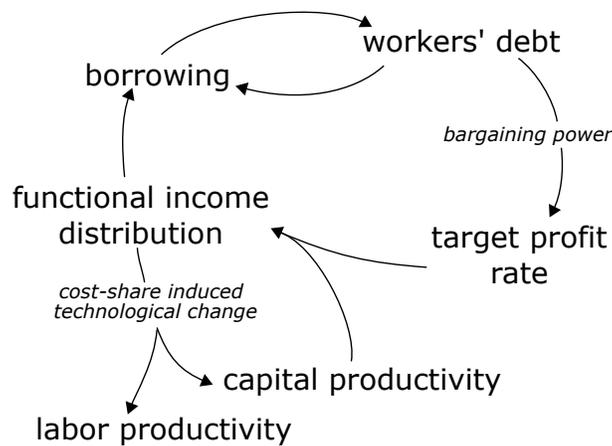


Figure 4: Causal links between debt, distribution and productivity

## 2 Model

The model includes workers' and investors' households, and firms. Profits, including debt service paid by workers, may be either saved or consumed, and the amount saved may be either invested or loaned to workers. This is a single-sector model with a common price level  $P$ . We work with real variables and in continuous time throughout.

## 2.1 Firms

Firms produce both consumption and investment goods using their capital stock  $K$ . They target a normal capital utilization rate, which we assume to be fixed. For notational simplicity, we define capital productivity  $\kappa$  as output  $Y^*$  at normal utilization divided by the value of the capital stock, and the utilization factor  $u$  as capital utilization relative to the normal rate. Thus,  $u$  can exceed a value of one, with  $u = 1$  at normal utilization,

$$Y = uY^* = u\kappa K. \quad (1)$$

We assume that output is constrained by the availability of capital, but not labor, and that workers in all positions are laid off and re-hired as utilization falls below or rises above the normal level. Denoting labor productivity at normal utilization by  $\lambda$ , employment  $L$  is then given by

$$L = u \frac{Y^*}{\lambda} = \frac{Y}{\lambda}. \quad (2)$$

Workers are paid a unit wage  $w$ , for a total wage bill  $W = wL$ .

Firms apply a profit margin  $\mu$  to normal labor costs. Setting  $u = 1$  in equation (2), the unit price of goods,  $P$ , is

$$P = \mu \frac{w}{\lambda}. \quad (3)$$

Profits  $\Pi$  are given by revenues net of wages,

$$\Pi = PY - W = (\mu - 1)W. \quad (4)$$

The profit margin is thus related to the profit and wage shares through

$$\mu = \frac{1}{1 - \pi} = \frac{1}{\psi}. \quad (5)$$

This result follows from the flexible employment assumption. We could have assumed instead that only production workers are on flexible contracts, while administrative, managerial and support staffing levels are maintained at a steady rate over the business cycle. In that case, profits would vary pro-cyclically. While more realistic, such an assumption would add complexity without insight.

The gross profit rate,  $r$ , at normal utilization is the product of the profit share  $\pi$  and the capital productivity at normal utilization  $\kappa$ . We assume that firms set their markup to meet an exogenous target rate of return at normal utilization  $r^*$ , so the profit and wage shares are given by

$$\pi = \frac{r^*}{\kappa}, \quad \psi = 1 - \pi. \quad (6)$$

We focus on net new investment  $I$  and do not represent depreciation explicitly. The form for the investment function is a contested subject (see Lavoie (1995)), but it does not play a crucial role in the current paper. Fazzari and Mott (1986) found evidence that utilization and internal finance generated from profits were both significant explanatory variables for firm investment behavior (and a negative influence from interest expense). Because firms in our model are able to set prices and wages so as to achieve a target profit rate, we propose that the net investment function  $g_i$  responds to utilization,

$$g_i \equiv \frac{I}{K} = \gamma + \alpha u. \quad (7)$$

Capital stock dynamics then become,

$$\dot{K} = I = g_i K. \quad (8)$$

## 2.2 Investor households

Investor households receive profits from firms and interest payments from worker households. Out of this they save a fraction  $s_i$ , and of the saved fraction they meet the net borrowing requirements of worker households and supply savings for firm investment. Denoting the stock of worker debt by  $D_W$  and nominal interest by  $i$ , total income  $\Pi$  for investor households is

$$\Pi = \pi Y + (i - \hat{P}) D_W = \pi \kappa u K + (i - \hat{P}) D_W. \quad (9)$$

The net saving available to firms,  $S$ , is then

$$S = s_i \Pi - \dot{D}_W = s_i \pi \kappa u K + s_i (i - \hat{P}) D_W - \dot{D}_W. \quad (10)$$

Note that workers may save, in which case their net new borrowing  $\dot{D}_W$  will be negative.

We normalize all variables by the capital stock, using lower-case letters for normalized variables. Note that

$$d_W = \frac{D_W}{K} \Rightarrow \dot{d}_W = \frac{\dot{D}_W}{K} - g_i d_W \Rightarrow \frac{\dot{D}_W}{K} = \dot{d}_W + g_i d_W. \quad (11)$$

This allows us to express equation (10) as

$$g_s \equiv \frac{S}{K} = s_i \pi \kappa u + s_i (i - \hat{P}) d_W - (\dot{d}_W + g_i d_W). \quad (12)$$

In this closed economy, saving equals investment, or  $g_s = g_i \equiv g_K$ . Applying

this to the preceding equation gives

$$s_i \pi \kappa u + s_i (i - \hat{P}) d_W - (\dot{d}_W + g_K d_W) = g_K. \quad (13)$$

This equation in turn can be rearranged to distinguish leakage and injection terms for the economy,

$$s_i \pi \kappa u + s_i (i - \hat{P}) d_W = \dot{d}_W + g_K d_W + g_K. \quad (14)$$

The left hand side of this equation is leakage as it denotes saving by investor households. The right hand side is injection to the economy as it denotes investment and lending to (and borrowing by) workers, who will use the borrowed funds to finance consumption.

### 2.3 Worker households

Worker households consume an amount  $C_W$  out of wages  $W$  and net borrowing. If their consumption and debt service is less than their wage, then they save. Their consumption is given by

$$C_W = W - (i - \hat{P}) D_W + \dot{D}_W. \quad (15)$$

The term  $W - (i - \hat{P}) D_W$  denotes disposable income after interest payments corrected for inflation. The rate of change of debt,  $\dot{D}_W$ , is net new borrowing. Normalized by the capital stock, we find:

$$c_W = \psi \kappa u - (i - \hat{P}) d_W + \dot{d}_W + g_K d_W. \quad (16)$$

Workers' debt-financed consumption behavior plays a pivotal role in the analysis to follow. We use the following behavioral function to describe this

behavior:

$$\dot{d}_W = f(d_W, \psi). \quad (17)$$

Equation (17) suggests that workers' borrowing depends on their previously accumulated debt and their wage income. Note that equation (17) specifies workers' *normalized* borrowing behavior, rather than their absolute level of borrowing ( $\dot{D}_W$ ) in any period.

There are various combinations of economically reasonable behavior that might sign the partial derivatives of the function in equation (17), and so describe the precise manner in which working households approach the process of debt accumulation. Given the trend of a falling wage share from the 1990s, we first consider the case in which the wage share falls. If wages do not keep pace with costs, then, for worker households, a fall in the wage share can translate into increased indebtedness ( $f_\psi < 0$ ); alternatively, they may reduce their borrowing as they receive higher wage share ( $f_\psi < 0$ ) if their borrowing is motivated principally by an exogenous target level of consumption, more of which can now be funded by current income. On the other hand, if worker households can meet their living expenses without undue stress when wage share falls, then they may be able to reduce their indebtedness ( $f_\psi > 0$ )

Workers' borrowing may increase with rising indebtedness ( $f_{d_W} > 0$ ), since more debt increases the debt-servicing burden and hence the need to borrow to achieve a level of consumption, whether that level is determined by the household's own preferences or is imposed on them by the cost of living. This behavior would result in the debt-fueled consumption that has been characteristic of financialization. If, instead, working households are both able and willing to behave more frugally, they may reduce their borrowing in response to an increase in their indebtedness ( $f_{d_W} < 0$ ) to avoid exceeding an exogenous target debt level.

As this brief discussion reveals, the description of borrowing behavior in (17) ultimately allows for partial derivatives of differing signs depending on whether workers' borrowing behavior is principally driven by consumption considerations (such as an exogenous target level of consumption) or financial considerations (such as an exogenous target level of debt) (see, for example, Setterfield and Kim (2016) and Dutt (2005, 2006), respectively). Note also that the larger the magnitude of the partial derivatives of equation (17), the quicker will be the adjustment of workers' borrowing (and consumption) behavior to changes in their indebtedness and the wage share. As will become clear, the magnitudes (as well as the signs) of the partial derivatives of (17) play an important role in the economy's macrodynamics.

## **2.4 Cost-share induced technological change**

The proposition that cost saving drives technical change was familiar to the classical economists (Kurz, 2010). Marx made it a centerpiece of his critique of capitalism, noting that while capitalists could gain temporary excess profits after a cost-saving innovation, it would soon be copied, thereby eroding their profits. Competition under Marx-biased technological change thus drives profit rates downward. Modern theories of cost-induced technological change were inspired by Hicks (1932), who argued that firms would seek to raise productivity in the factor with the highest cost share. Absent exogenous changes in the relative prices of factors, the result is equal rates of marginal productivity growth for all factors of production, or Hicks-neutral technological change. The neoclassical assumption of Hicks-neutral technological change was criticized by Kaldor (1961, pp. 203-207) as being inconsistent with the (stylized) facts, among them that the capital-output ratio, wage and profit shares, and the profit rate, are all roughly constant, while labor productivity has grown

substantially. Kaldor's stylized facts met the criteria for Harrod-neutral technological change, in which labor productivity grows, while capital productivity is constant. Kaldor's observations have been criticized in their turn, because capital-output ratios have been shown to vary over a wide but bounded range over time, while wage and profit shares have both risen and fallen over decadal time scales. Further well-known contributions to the theory of induced technological change along neoclassical lines were put forward by Kennedy (1964), Samuelson (1965), and Acemoglu (2002).

Less well-known is a classical-Marxian evolutionary model of technological change proposed by Duménil and Lévy (1995, 2010). In evolutionary theory, firms seek marginal improvements on their current practice in an essentially random manner (Nelson and Winter, 1982). Duménil and Lévy formalized this idea by supposing that firms seek labor and capital productivity-enhancing innovations with an unbiased probability distribution. That is, they are just as likely to discover an innovation that would raise capital productivity by 0.0% and labor productivity by 0.5% as the other way around. However, firms only accept those innovations that would raise their profit rate at constant wages and prices – this is the Okishio (1961) viability criterion. This imparts a bias toward technological progress, even though the probability of discovery is unbiased. Innovation gives firms a temporary monopoly, and by assumption raises the rate of profit. However, in keeping with Marx's observation that competition erodes whatever temporary profits the firm might achieve, after innovations diffuse among firms, productivity growth induces a round of wage and price setting that can result in a higher, lower, or unchanged rate of profit.

The theory of Duménil and Lévy is promising, but plagued by difficulties: their model is limited to wage and profit shares; the unbiased (and, indeed, circularly symmetric) probability distribution for innovations is a severe limitation

(Foley, 2003); and the resulting theory offers little guidance for candidate functional forms relating cost shares and productivity growth rates (Julius, 2005). Kemp-Benedict (in press) showed that these limitations can be overcome. With no restrictions on the number of inputs, and requiring only that the probability distribution of innovations not depend on cost shares, he found that the combination of random search with Okishio's viability criterion implied specific restrictions (discussed below) on the relationship between cost shares and productivity growth rates. We apply that theory in this paper.

In any cost-share induced model of technological change, capital and labor productivity growth depends on the functional income distribution. In this paper, there are two inputs – labor and capital – with corresponding cost shares  $\psi$  and  $\pi$ . The cost shares are not independent, because they sum to one. However, the constraints on the cost share-productivity growth rate relationship discussed below are clearest when wage and profit shares are treated initially as independent. We therefore defer imposing the condition that they sum to one until later in the analysis, and write

$$\hat{\kappa} = k(\pi, \psi), \tag{18a}$$

$$\hat{\lambda} = \ell(\pi, \psi). \tag{18b}$$

Kemp-Benedict (in press) showed that, under the assumptions listed above, the matrix  $M$  of partial derivatives

$$M = \begin{pmatrix} k_{\pi} & k_{\psi} \\ \ell_{\pi} & \ell_{\psi} \end{pmatrix} \tag{19}$$

is symmetric and positive semi-definite, with a null vector made up of the cost shares  $(\pi, \psi)$ . This finding addresses the concern that Duménil and Lévy's the-

ory gives little guidance on candidate functional forms (Julius, 2005). Positivity and symmetry imply that  $k_\pi, \ell_\psi > 0$  and  $k_\psi = \ell_\pi$ . The requirement that the cost shares form a null vector implies that the off-diagonal elements are negative, as we now show. We first write the null vector condition,

$$\pi k_\pi + \psi k_\psi = \psi \ell_\psi + \pi \ell_\pi = 0. \quad (20)$$

Taking the first of these expressions and solving for  $k_\psi$ ,

$$k_\psi = -\frac{\pi}{\psi} k_\pi. \quad (21)$$

Because  $k_\pi > 0$  from the positivity condition, and the cost shares are also positive, the off-diagonal term  $k_\psi$  must be negative. This result follows from a trade-off implicit in the Okishio viability criterion. When the cost share of one input rises, it promotes greater productivity in the use of that input as firms seek temporary monopoly profits. The direct effect on the other input is to retard productivity growth.

We now impose the condition that the cost shares sum to one, and define a function for  $\hat{\kappa}$  that depends only on  $\psi$ ,<sup>3</sup>

$$\hat{\kappa} = \rho(\psi) \equiv k(1 - \psi, \psi), \quad \rho'(\psi) = \left(1 + \frac{\psi}{\pi}\right) k_\psi < 0. \quad (22)$$

The result that  $\hat{\kappa}$  is inversely related to  $\psi$  applies to innovation within firms. After innovations diffuse, firms adjust prices and wages.

In this paper, we assume target return pricing (Lavoie, 2016), in which firms adjust their markup  $\mu$ , and therefore the wage share  $\psi$ , to ensure that the profit

---

<sup>3</sup>Note, from equation (22), that  $\rho' = k_\pi(-1) + k_\psi(+1)$ . From equation (21) we have  $k_\pi = -(\psi/\pi)k_\psi$ . Together, these expressions yield  $\rho' = (1 + \psi/\pi)k_\psi$ . Because  $k_\psi$  is negative, so is  $\rho'$ , so the capital productivity growth rate responds negatively to a rise in the wage share. By a similar argument we can show that the labor productivity growth rate responds positively to a rise in the wage share.

rate is equal to a target rate  $r^*$ , as shown in equation (6).<sup>4</sup> Thus,

$$\pi\kappa = (1 - \psi)\kappa = r^*. \quad (23)$$

Taking the time derivative of each side of this equation and rearranging, we find an expression for the time rate of change of  $\psi$ ,

$$\dot{\psi} = (1 - \psi)(\hat{\kappa} - \hat{r}^*) = (1 - \psi)(\rho(\psi) - \hat{r}^*). \quad (24)$$

We can combine this equation with equation (22) to find

$$\frac{d\hat{\kappa}}{dt} = \rho'(\psi)(1 - \psi)(\hat{\kappa} - \hat{r}^*). \quad (25)$$

The coefficient on  $\hat{\kappa}$  is negative because  $\rho'(\psi)$  is negative, so this is a stabilizing dynamic that tends to drive the capital productivity growth rate towards  $\hat{\kappa} = \hat{r}^*$ . If the target profit rate is not changing, then capital productivity is a constant at equilibrium. From equation (22), this means that the wage share  $\psi$  is also constant at equilibrium, taking the value at which  $\rho(\psi) = 0$ .

## 2.5 Household debt and distribution

Based on the bargaining power and debt argument, the profit rate is an increasing function of workers' indebtedness,

$$r^* = r^*(d_W), \quad r^{*'} > 0. \quad (26)$$

Our specification reflects the idea that, as workers are more indebted, the cost of losing their job becomes greater as they will have to make debt service payments.

---

<sup>4</sup>If, instead, firms were to adopt a fixed markup, then  $\psi$  would be constant. From equation (22),  $\hat{\kappa}$  must also be constant, and if it takes a negative value, the result is Marx-biased technological change.

Therefore, their bargaining power is reduced, allowing the firms to target a higher profit rate. Our argument is grounded in a number of recent observations to the effect that rising household indebtedness disempowers workers and affects the wage bargain.

Bryan et al. (2009, p.13) point out that consumer credit involves the rendering of surplus value to workers prior to production, thus intensifying workers' commitment to the production system. Bonefeld (1995, p.69) argues that debt undermines workers' resistance to wage reductions and the intensification of work. According to Slater and Spencer (2014, p.142), financialization, as reflected in the accumulation of financial assets and accumulation of personal debt, have increased the power of employers over their workers. These claims are in line with the findings of Karacimen (2015), whose questionnaires and interviews with workers in the Turkish metal working sector reveal that indebtedness transforms capital-labor relations, ultimately increasing workers' sense of commitment to their employers. Similarly, Butrica and Karamcheva (2014) report that older adults with debt in the US are 8 percentage points more likely to work and 2 percentage points less likely to receive Social Security benefits than those without debt, suggesting that indebtedness forces older individuals to keep working.

Comparing to equation (24), we see that the growth rate of the target rate of return enters into the expression for the growth rate of the wage share  $\psi$ . From equation (26), this is equal to

$$\hat{r}^*(d_W) = \frac{r^{*'}}{r^*} \dot{d}_W \equiv \varphi(d_W) \dot{d}_W, \quad (27)$$

where  $\varphi(d_W)$  takes on positive values. From equation (17), this is seen to equal

$$\hat{r}^*(d_W) = \varphi(d_W) f(d_W, \psi). \quad (28)$$

When multiplied by  $d_W$ , the function  $\varphi(d_W)$  becomes the elasticity of the target profit rate to household indebtedness. It therefore measures the responsiveness of firms' price- and wage-setting strategy to the debt held by their workers. While we do not pursue a full empirical analysis in this paper, there is some evidence that  $\varphi(d_W)$  became positive starting in the 1990s.<sup>5</sup>

## 2.6 Dynamic system

From the foregoing, we have the following system of equations:

$$\dot{d}_W = f(d_W, \psi), \quad (29a)$$

$$\dot{\psi} = h(d_W, \psi), \quad (29b)$$

where

$$h(d_W, \psi) \equiv (1 - \psi) (\rho(\psi) - \varphi(d_W) f(d_W, \psi)). \quad (30)$$

The equilibrium for this dynamic system satisfies  $f(d_W, \psi) = h(d_W, \psi) = 0$ .

From this expression we see that at equilibrium

$$h(d_W, \psi) = 0 \Rightarrow \rho(\psi) = \varphi(d_W) f(d_W, \psi) = 0. \quad (31)$$

From equation (22), when  $\rho(\psi)$  is zero, capital productivity is not changing, so the equilibrium satisfies at least two of Kaldor's stylized facts: constant wage share and constant capital productivity.

We next consider local stability conditions in a small neighborhood around

---

<sup>5</sup>From 1967 to 1989, the log of the profit rate shown in figure 3 was of a lower order of integration than the debt-to-capital stock ratio shown in figure 2, suggesting additional influences on indebtedness, such as changing social norms (Cynamon and Fazzari, 2008) or income inequality (Barba and Pivetti, 2009). Between 1990 and 2007, both series (and the series for the wage share) were integrated of order one. Applying the Toda-Yamamoto procedure (Toda and Yamamoto, 1995), which is an extension of the Granger test to non-stationary time series, for the later period, we found evidence of Granger causality from  $d_W$  to  $\log(r)$  ( $p = 0.036$ ), but not *vice versa* ( $p = 0.31$ ). We also find evidence of cointegration for these two variables for this time period. Thus, profit rates appeared to respond to household debt after 1990.

the equilibrium. Denoting deviations from equilibrium by  $\Delta d_W$  and  $\Delta\psi$ , we have

$$\Delta \dot{d}_W = f_{d_W} \Delta d_W + f_\psi \Delta\psi, \quad (32a)$$

$$\Delta \dot{\psi} = h_d \Delta d_W + h_\psi \Delta\psi. \quad (32b)$$

From the definition of  $h(d_W, \psi)$  in equation (30), its partial derivatives evaluated at the equilibrium are given by

$$h_d = -(1 - \psi) \varphi(d_W) f_{d_W}, \quad (33a)$$

$$h_\psi = (1 - \psi) (\rho'(\psi) - \varphi(d_W) f_\psi). \quad (33b)$$

The criterion for local stability is that the Jacobian in equations (32) must have a negative trace and a positive determinant. The trace  $\text{Tr}$  is given by

$$\text{Tr} = f_{d_W} + h_\psi = f_{d_W} + (1 - \psi) (\rho'(\psi) - \varphi(d_W) f_\psi). \quad (34)$$

The determinant  $\text{Det}$  is given by

$$\text{Det} = f_{d_W} h_\psi - f_\psi h_d = f_{d_W} (1 - \psi) \rho'(\psi). \quad (35)$$

From the expression for the determinant, and noting that  $\rho'(\psi)$  is negative, a necessary condition for stability is  $f_{d_W} < 0$ . This makes sense, as it says that debt should not experience accelerating dynamics.

When  $f_{d_W} < 0$ , the trace will certainly be negative if  $f_\psi > 0$ ; that is, if households tend to reduce their debt when the wage share decreases. However, it is not a necessary condition. It is possible for  $f_\psi < 0$  and still have a negative

trace as long as  $f_\psi$  exceeds a critical level

$$f_\psi > \frac{f_{d_W} + (1 - \psi) \rho'(\psi)}{(1 - \psi) \varphi(d_W)} \equiv f_\psi^{\text{crit}}(d_W, \psi). \quad (36)$$

Note that the constraint becomes less binding the smaller the denominator. The numerator is negative, so as the denominator gets smaller the entire expression becomes more negative.

The responsiveness of firms to household debt,  $\varphi(d_W)$ , plays a crucial role in determining the stability of the system. If profit rates are insensitive to debt levels, then the denominator in equation (36) is small and the debt-wage share equilibrium can be locally stable even if  $f_\psi$  is large and negative. However, if rates of return are sensitive to household debt levels, as may have been the case starting in the 1990s (see footnote 5) then a negative value for  $f_\psi$  could make the debt-wage share system unstable.

### 3 Analysis

With the model in hand, we next consider its relevance for the historical US economy and its implications for the future. First, we examine the historical data plotted in figures 2 and 3 to better understand how the factors have changed over time. Then, we use the dynamical analysis in the previous section to examine debt and distributional regimes.

#### 3.1 Observations of time series

The main source of evidence for this paper is critical analysis of the US economy from a political economy perspective, rather than econometric analysis. Nevertheless, we can gain some preliminary empirical insights from the time series of

$d_W$ ,  $r$ , and  $\psi$  plotted in figures 2 and 3.<sup>6</sup>

The trend in the household debt-to-capital ratio shown in figure 2 has noticeable breaks. We tested for breaks, imposing a minimum of 11 years between them, equal to twice the 69-month average length of postwar business cycles. The minimum Bayes Information Criterion (BIC) score was found for three break dates, at 1966, 1985, and 1996. Interestingly, the wage share had similar break dates, in 1968, 1985, and 1996. The rate of change of the wage share was slowest in the periods 1952-1968 (-0.08%/year) and 1986-1996 (-0.05%/year), while between 1969 and 1985, it declined at 0.31%/year and after 1996 at 0.19%/year. The correspondence between the break dates suggests a link between income distribution and debt, consistent with the findings of Barba and Pivetti (2009).

Applying these break dates to the time series gives periods too short for reliable analysis, so as noted in footnote 5, we studied the periods 1967-1989 and 1990-2007, ending the analysis before the onset of financial crisis. In the earlier time period, the  $d_W$  time series was of a different order of integration than the other two series, suggesting that other factors were active. This makes sense, as both norms and opportunities for taking on debt were evolving over the period. The issuance of “revolving credit” through credit cards was still novel in 1966 (Zumello, 2011), and the normalization of indebtedness did not take hold until the 1980s (Cynamon and Fazzari, 2008). After 1989,  $d_W$ ,  $r$ , and  $\psi$  are all integrated of order one.

We found evidence that household debt was a leading and positive indicator of the profit rate between 1990 and 2007, suggesting that  $\varphi(d_W)$  was positive in this period; see footnote 5 for details. As a naive check on the debt dynamics in

---

<sup>6</sup>All tests were carried out in R using packages VARS for VAR models and related tests (Pfaff, 2008b) and urca for unit root tests (Pfaff, 2008a). Wald tests were carried out using the aod package (Lesnoff et al., 2012), while structural change tests were carried out using the strucchange package (Zeileis et al., 2003).

equation (17), we regressed the first difference of  $d_W$  against one-period lagged values of  $d_W$  and  $\psi$ . For the 1990-2007 period, we found that  $\dot{d}_W$  was related positively and significantly to  $d_W$  ( $p < 0.001$ ) and positively but not significantly to  $\psi$  ( $p = 0.13$ ). Thus, for the post-1989 period we find some evidence that  $f_{d_W}$  was positive.

### 3.2 Debt and distributional regimes

In the model, workers' borrowing behavior plays a decisive role in determining whether or not the system is stable, a consequence of the endogeneity of borrowing and bargaining (equation (26)) in terms of household debt and the wage share.

Debt and distributional dynamics in our model are certainly unstable when debt breeds more debt in an accelerating spiral ( $f_{d_W} > 0$ ). In other words, as long as workers are excessive in borrowing in the sense that they borrow more for consumption when their indebtedness increases, the system is unstable. From the brief empirical analysis in the previous section, we found evidence that  $f_{d_W}$  was positive in the 1990-2007 period. As noted earlier, this was a period of debt-fuelled consumption in the US, an unstable expansion that collapsed in the 2007-8 financial crisis (Guttman and Plihon, 2010). We therefore find that the model, at least in this unstable regime, is consistent with historical experience.

Next, we consider the case  $f_{d_W} < 0$  and ask what might lead to stable or unstable outcomes. At equilibrium,  $f(d_W, \psi) = 0$  and therefore, from equation (31),  $\rho(\psi) = 0$  as well. We assume that at a high wage share working households are able to accommodate their debt levels to their incomes, so  $f_\psi > 0$  at high values of  $\psi$ . When the wage share is low, households face difficulties meeting needed expenses, so  $f_\psi < 0$  at low values of  $\psi$ . These cases are illustrated in

Figure 5 for three possible equilibrium wage shares, determined by  $\rho_i(\psi) = 0$ , for  $i = 1, 2, 3$ .

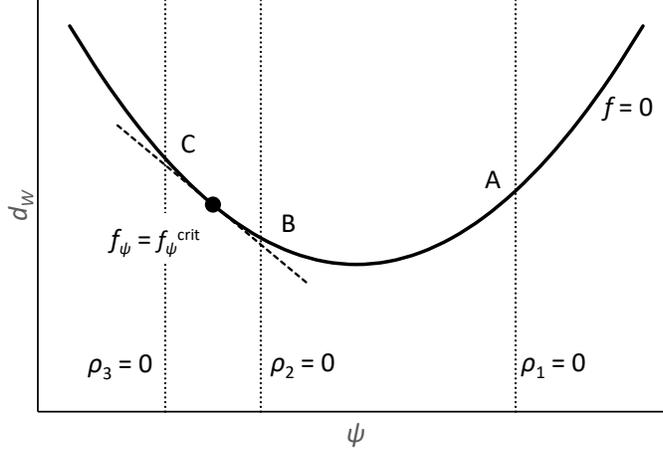


Figure 5: Equilibria for  $\psi$ -dependent debt dynamics and different expressions for  $\rho$

- **Case A in Figure 5:**  $f_{d_W} < 0$  and  $f_\psi > 0$ , where we see workers reduce their borrowing when their indebtedness increases, but they are more willing to borrow more when their income share increases. In this case, we observe stable wage and profit shares as well as stable capital and labor productivity growth.
- **Case B in Figure 5:**  $f_{d_W} < 0$  and  $f_\psi < 0$  and  $f_\psi$  satisfies the condition of  $f_\psi > f_\psi^{\text{crit}}(d_W, \psi)$ . We see workers reduce their borrowing when their indebtedness increases, and they borrow less when their incomes increase. In this case, again, we observe stable wage and profit shares as well as stable capital and labor productivity growth.
- **3. Case C in Figure 5:**  $f_{d_W} < 0$  and  $f_\psi < 0$  and  $f_\psi$  does *not* satisfy the condition of  $f_\psi > f_\psi^{\text{crit}}(d_W, \psi)$ . In this case, the equilibrium is unstable;

a deviation toward lower wage share will lead to a self-reinforcing spiral of falling wage share and rising debt, as seen from figure 5.

In Case A, workers' debt is entirely discretionary. Worker households reduce their debt levels if either their debt rises relative to the existing capital stock or if their income share falls. This flexibility also enhances their negotiating power. While firms are free to set prices to achieve a target rate of return, they are not free to raise the target rate as much as they might wish to, because they must share any gains with workers. While these conditions are superficially similar to the 1952-1965 period, the causal links at that time were different than those shown in figure 4, because bargaining power was exercised directly by organized labor, not indirectly through greater or lesser levels of debt.

Case B might have characterized the decade from 1986-1996. As can be seen in figure 2, both the wage share and household indebtedness appear to have reached a plateau in that period, although debt appears to have a great deal of inertia, reflected in a higher order of integration. During this period, the change in  $d_W$  was negatively related to the lagged value of  $\psi$  (although marginally:  $p = 0.075$ ), suggesting that  $f_\psi$  was negative.

Case C is one of two unstable cases we have identified. The other unstable case, in which  $f_{d_W} > 0$ , leads to self-reinforcing debt-fueled consumption, as observed starting in the 1990s. The situation in Case C is different. Here, worker households cannot readily reduce consumption when the wage share falls. As with households in income poverty, they may then experience hardship (Iceland and Bauman, 2007), or avoid hardship by incurring debt (Beverly, 2001). In either case, their vulnerability weakens their bargaining power. Case C thus features a self-reinforcing cycle of rising inequality and debt. The opposite case can happen as well, at least within the model, in that a rising wage share could allow worker households to reduce their debt and demand yet higher wages.

Regardless of the direction of change, at some point the explosive trend will be stopped by a process that lies outside the model, as the 2008 financial crisis stopped the trend of exponentially accumulating debt.

Thus, in Case C, growth in the wage share can trigger a virtuous cycle of falling debt and rising wage share, but a fall in the wage share can trigger rising debt and a continually falling wage share. A critical parameter is  $\varphi(d_W)$ , which captures how strongly workers' indebtedness affect their bargaining strength and hence the functional income distribution. It plays an important role for determining the stability of the system when  $f_\psi < 0$ . When it is high, the critical value  $f_\psi^{\text{crit}}(d_W, \psi)$  is less negative, and the point at which  $f_\psi = f_\psi^{\text{crit}}$  in Figure 5 shifts rightward. In other words, the stable region of the economy shrinks as it becomes unstable at a lower value of  $d_W$  and a higher value of  $\psi$ .

## 4 Discussion

The US economy experienced profound changes in the second half of the 20th century. Household debt rose throughout the postwar era, driven first by a boom and later by a bubble. The labor protections put in place in the Great Depression and the Second World War were gradually weakened, and union membership declined, a process that accelerated sharply from the late 1970s. Capital productivity fell in the postwar recession, but then recovered, reaching a peak in the mid-1960s, and then generally falling through the 1980s. As the profit share was comparatively steady through the mid-1970s, this trend was initially accompanied by a falling profit rate, as shown in figure 3.

The early to mid-1970s thus featured meaningful, albeit weakening, labor bargaining power combined with a falling profit rate. This is the situation posited by Marx, in which firms innovate to raise their profits, only to see those profits undercut through competition and demands from labor for a share of

the benefits. However, *contra* Marx, the profit rate did not continue to fall. Firms became more active in undermining labor power, and after 1980, the government increasingly supported their efforts. This freed firms from the need to share the benefits of innovation with workers and instead first stabilize and then raise their profit rates.

As the wage share fell, household debt began to rise. Before the 1990s, that rise was contained, as debt accumulation responded negatively to a rising indebtedness, and the profit rate was decoupled from debt. However, in the 1990-2007 period, the profit rate was responsive to household debt, indicating significantly weakened labor bargaining power. During the same period, debt accumulation became more responsive to the debt level, driving a spiral of rising indebtedness that ended with the 2007-8 financial crisis.

The model could be extended in several directions. We have proposed saving and investment behavior, which invites a Kaleckian analysis of utilization dynamics. As utilization depends on the functional income distribution, which in our model depends on debt, the model implicitly links debt to utilization. The link depends on the relationship between utilization and wage income, which is affected by the labor market closure. A different closure – for example, one in which a permanent staff is maintained over the business cycle – would lead to a different relationship between utilization and the functional income distribution. The link between utilization and the functional income distribution also depends on household saving behavior. The shape of the  $f = 0$  nullcline in figure 5 assumes that workers’ households face tighter constraints on saving when income is distributed more unequally. This assumption could be put on a formal footing by modifying the consumption function.

Finally, the model can provide some insights on the possibility of “secular stagnation”. While secular stagnation has historically been a topic within

heterodox economics (Steindl, 1952, 1979; Hein, 2015), it has recently been brought into prominence in the mainstream literature (Summers, 2014; Teulings and Baldwin, 2014) and is part of an ongoing and highly visible debate. The connection to the model is through cost-share induced technological change. Technological paradigms succeed each other in waves, and at the end of each wave, it becomes increasingly difficult to raise productivity with the incumbent technology, creating space for another to expand (Perez, 2002). As argued by Gordon (2016), the 1970s may have marked the end of such a wave. Within the model,  $\hat{\kappa} = \rho(\psi) = 0$  at equilibrium. If, in the course of a wave of technological development,  $\hat{\kappa}$  were to fall, so that  $\rho(\psi)$  becomes negative, then  $\rho$  would have to increase for a new equilibrium to be established. As  $\rho$  depends inversely on  $\psi$ , this means that the new equilibrium must be at a lower value of  $\psi$ . Thus, we expect the  $\rho = 0$  nullcline in figure 5 to gradually move leftward, from  $\rho_1$  to  $\rho_2$  and then  $\rho_3$  unless stopped by the appearance of a new disruptive technology. Unstable debt dynamics in the region of the Case C equilibrium can depress innovation and growth, as the fall in the wage share depresses labor productivity growth and promotes capital productivity growth, thus slowing the pace of capital deepening.

## 5 Conclusion

We have presented a model of coupled household debt-productivity growth-distribution dynamics. The dynamical system can be either stable or unstable, with the potential for a self-reinforcing pattern of rising household indebtedness and falling wage share, consistent with trends in the US from the 1980s onward. The unstable cycle can be triggered by either an increasing willingness on the part of households to take on debt, as seen from the 1990s, and/or rising influence of household indebtedness on workers bargaining strength and income

distribution in the wave of financialization. The model thus sheds some light on widely-observed trends in recent decades in the US, and potentially in other OECD countries.

## References

- Acemoglu D. (2002) ‘Directed technical change’, *The Review of Economic Studies*, 69(4), pp. 781–809.  
**URL:** <http://restud.oxfordjournals.org.ezp.sub.su.se/content/69/4/781>
- Barba A., Pivetti M. (2009) ‘Rising household debt: Its causes and macroeconomic implications—a long-period analysis’, *Cambridge Journal of Economics*, 33(1), pp. 113–137.  
**URL:** <http://academic.oup.com/cje/article/33/1/113/1700026>
- Beverly S. G. (2001) ‘Measures of material hardship’, *Journal of Poverty*, 5(1), pp. 23–41.
- Bonefeld W. (1995) ‘The politics of debt: social discipline and control’, *Common Sense: Journal of the Conference of Socialist Economists*, 17, pp. 69–91.
- Bryan D., Martin R., Rafferty M. (2009) ‘Financialization and Marx: Giving labor and capital a financial makeover’, *Review of Radical Political Economics*, 41(4), pp. 458–472.  
**URL:** <http://EconPapers.repec.org/RePEc:sae:reorpe:v:41:y:2009:i:4:p:458-472>
- Butrica B. A., Karamcheva N. S. (2014) ‘Does household debt influence the labor supply and benefit claiming decisions of older americans?’, *Technical report*, Netspar Discussion Paper No. 03/2014-083, <http://dx.doi.org/10.2139/ssrn.2574724>.

- Cynamon B. Z., Fazzari S. M. (2008) 'Household debt in the consumer age: source of growth–risk of collapse', *Capitalism and Society*, 3(2).  
**URL:** <http://www.degruyter.com/view/j/cas.2008.3.2/cas.2008.3.2.1037/cas.2008.3.2.1037.xml>
- Duménil G., Lévy D. (1995) 'A stochastic model of technical change: An application to the US economy (1869-1989)', *Metroeconomica*, 46(3), pp. 213–245.  
**URL:** <http://onlinelibrary.wiley.com/doi/10.1111/j.1467-999X.1995.tb00380.x/abstract>
- Duménil G., Lévy D. (2010) 'The classical-Marxian evolutionary theory of technological change: application to historical tendencies', in Setterfield M. (ed.): *Handbook of Alternative Theories of Economic Growth*, Edward Elgar, Cheltenham, U.K. ; Northampton, Mass., USA, pp. 243–273.
- Dutt A. K. (2005) 'Consumption, debt and growth', in Setterfield M. (ed.): *Interactions in Analytical Political Economy*, M.E. Sharpe, Armonk, NY, pp. 155–78.
- Dutt A. K. (2006) 'Maturity, stagnation and consumer debt: a Steindlian approach', *Metroeconomica*, 57, pp. 339–364.
- Fazzari S. M., Mott T. L. (1986) 'The investment theories of Kalecki and Keynes: an empirical study of firm data, 1970-1982', *Journal of Post Keynesian Economics*, 9(2), pp. 171–187.  
**URL:** <http://www.tandfonline.com/doi/pdf/10.1080/01603477.1986.11489611>
- Feenstra R. C., Inklaar R., Timmer M. (2013) 'The next generation of the Penn World Table', *Working Paper 19255*, National Bureau of Economic Research.  
**URL:** <http://www.nber.org/papers/w19255>

- Foley D. K. (2003) 'Endogenous technical change with externalities in a classical growth model', *Journal of Economic Behavior & Organization*, 52(2), pp. 167–189.  
**URL:** <http://www.sciencedirect.com/science/article/pii/S0167268103000209>
- Foster J. B. (2007) 'The financialization of capitalism', *Monthly Review; New York*, 58(11), pp. 1–12.  
**URL:** <https://search.proquest.com/docview/213214866/abstract/C5AB152A6E354C3EPQ/1>
- Gordon R. J. (2016) *The Rise and Fall of American Growth: The U.S. Standard of Living Since the Civil War*, The Princeton economic history of the Western world, Princeton University Press, Princeton OCLC: 908084072.
- Guttman R., Plihon D. (2010) 'Consumer debt and financial fragility', *International Review of Applied Economics*, 24(3), pp. 269–283.  
**URL:** <https://doi.org/10.1080/02692171003701420>
- Hein E. (2013) 'Finance-dominated capitalism and re-distribution of income: A Kaleckian perspective', *Cambridge Journal of Economics*, advance online access.  
**URL:** <http://cje.oxfordjournals.org.ezproxy.library.tufts.edu/content/early/2013/08/08/cje.bet038>
- Hein E. (2015) 'Secular stagnation or stagnation policy? Steindl after Summers', *Working Paper 846*, Levy Economics Institute of Bard College, Annandale-on-Hudson, NY, US.  
**URL:** <http://www.levyinstitute.org/pubs/wp846.pdf>
- Hicks J. (1932) *The Theory of Wages*, MacMillan and Company Limited, London.
- Iceland J., Bauman K. J. (2007) 'Income poverty and material hardship: How strong is the association?', *The Journal of Socio-Economics*, 36(3),

pp. 376–396.

**URL:** <http://www.sciencedirect.com/science/article/pii/S1053535706001119>

Julius A. J. (2005) ‘Steady-state growth and distribution with an endogenous direction of technical change’, *Metroeconomica*, 56(1), pp. 101–125.

**URL:** <http://onlinelibrary.wiley.com.ezproxy.library.tufts.edu/doi/10.1111/j.1467-999X.2005.00209.x/abstract>

Kaldor N. (1961) ‘Capital accumulation and economic growth’, in Lutz F., Hague D. (eds): *The Theory of Capital*, Macmillan and Co. Ltd., pp. 177–222.

Karacimen E. (2015) ‘Interlinkages between credit, debt and the labour market: evidence from Turkey’, *Cambridge Journal of Economics*, 39(3), pp. 751–767.

**URL:** + <http://dx.doi.org/10.1093/cje/beu016>

Kemp-Benedict E. (in press) ‘Cost share induced technological change and Kaldor’s stylized facts’, *Metroeconomica*, .

Kennedy C. (1964) ‘Induced bias in innovation and the theory of distribution’, *The Economic Journal*, 74(295), pp. 541–547.

**URL:** <http://www.jstor.org/stable/2228295>

Kim Y. K. (2013) ‘Household debt, financialization, and macroeconomic performance in the United States, 1951-2009’, *Journal of Post Keynesian Economics*, 35(4), pp. 675–694.

**URL:** <http://www.tandfonline.com/doi/abs/10.2753/PKE0160-3477350408>

Kurz H. D. (2010) ‘Technical progress, capital accumulation and income distribution in Classical economics: Adam Smith, David Ricardo and Karl Marx’, *The European Journal of the History of Economic Thought*, 17(5),

pp. 1183–1222.

**URL:** <http://dx.doi.org/10.1080/09672567.2010.522242>

Lavoie M. (1995) ‘The Kaleckian model of growth and distribution and its neo-Ricardian and neo-Marxian critiques’, *Cambridge Journal of Economics*, 19(6), pp. 789–818.

Lavoie M. (2016) ‘Frederic Lee and post-Keynesian pricing theory’, *Review of Political Economy*, 28(2), pp. 169–186.

**URL:** <http://dx.doi.org/10.1080/09538259.2016.1149375>

Lesnoff, M., Lancelot, R. (2012) *aod: Analysis of Overdispersed Data* R package version 1.3.

**URL:** <http://cran.r-project.org/package=aod>

Nelson R. R., Winter S. G. (1982) *An evolutionary theory of economic change*, Belknap Press of Harvard University Press, Cambridge, Mass.

Okishio N. (1961) ‘Technical changes and the rate of profit’, *Kobe University Economic Review*, 7, pp. 86–99.

Perez C. (2002) *Technological Revolutions and Financial Capital: The Dynamics of Bubbles and Golden Ages*, Edward Elgar Pub, Cheltenham.

Pfaff B. (2008a) *Analysis of integrated and cointegrated time series with R*, Springer, New York.

**URL:** <http://public.eblib.com/EBLPublic/PublicView.do?ptiID=364564>

Pfaff B. (2008b) ‘VAR, SVAR and SVEC models: Implementation within R package vars’, *Journal of Statistical Software*, 27(4), pp. 1–32.

**URL:** <http://vps.fmvz.usp.br/CRAN/web/packages/vars/vignettes/vars.pdf>

- Samuelson P. A. (1965) ‘A theory of induced innovation along Kennedy-Weisäcker lines’, *The Review of Economics and Statistics*, 47(4), pp. 343–356.  
**URL:** <http://www.jstor.org.ezproxy.library.tufts.edu/stable/1927763>
- Setterfield M., Kim Y. K. (2016) ‘Debt servicing, aggregate consumption, and growth’, *Structural Change and Economic Dynamics*, 36, pp. 22–33.
- Slater G., Spencer D. A. (2014) ‘Workplace relations, unemployment and finance-dominated capitalism’, *Review of Keynesian Economics*, 2(2), pp. 134 – 146.  
**URL:** [//www.elgaronline.com/journals/roke/2-2/roke.2014.02.01.xml](http://www.elgaronline.com/journals/roke/2-2/roke.2014.02.01.xml)
- Steindl J. (1952) *Maturity and stagnation in American capitalism*, number 4 in *Oxford University Institute of Statistics Monograph*, Basil Blackwell, Inc., Oxford, UK.
- Steindl J. (1979) ‘Stagnation theory and stagnation policy’, *Cambridge Journal of Economics*, 3(1), pp. 1–14.
- Summers L. H. (2014) ‘U.S. economic prospects: secular stagnation, hysteresis, and the zero lower bound’, *Business Economics*, 49(2), pp. 65–73.  
**URL:** <http://link.springer.com/article/10.1057/be.2014.13>
- Teulings C., Baldwin R. (eds) (2014) *Secular stagnation: facts, causes and cures*, Centre for Economic Policy Research, London, UK.
- Toda H. Y., Yamamoto T. (1995) ‘Statistical inference in vector autoregressions with possibly integrated processes’, *Journal of Econometrics*, 66(1-2), pp. 225–250.  
**URL:** <http://www.sciencedirect.com/science/article/pii/0304407694016168>

Zeileis A., Kleiber C., Krämer W., Hornik K. (2003) 'Testing and dating of structural changes in practice', *Computational Statistics & Data Analysis*, 44(1-2), pp. 109-123.

**URL:** <http://www.sciencedirect.com/science/article/pii/S0167947303000306>

Zumello C. (2011) 'The "Everything Card" and consumer credit in the United States in the 1960s', *The Business History Review*, 85(3), pp. 551-575.

**URL:** <http://www.jstor.org/stable/41301434>