Lost in Fiscal Space: Some Simple Analytics of Macroeconomic Policy in the Spirit of Tinbergen, Wicksell and Lerner

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Abstract

The interest rate and the fiscal balance can be thought of as two independent instruments to be assigned to two targets, the path of output and the path of public debt. Under what we term a 'sound finance rule' the interest rate targets output while the fiscal balance targets public debt; under a 'functional finance rule' the budget balance is assigned to the output gap and the interest rate to the debt ratio. The same unique combination of interest rate and fiscal balance will be consistent with output at potential and a constant debt-GDP ratio regardless of which instrument is assigned to which target The stability characteristics of the two rules differ, however. At low levels of debt, both rules converge, but at high levels of debt, only the functional finance rule converges. So contrary to conventional wisdom, the case for countercyclical fiscal policy becomes stronger, not weaker, when the ratio of public debt to GDP is already high. We apply our framework to describe policy generated cycles in the US over the past five decades.

Introduction

A central concern in recent debates over macroeconomic policy is the choice between the policy-determined interest rate and the government budget balance as tools for stabilizing output. A second major concern is the need to maintain the ratio of public debt to GDP on a "sustainable" trajectory. In this paper, we argue that these two issues must be addressed within a single framework, since both output and the public debt ratio are jointly determined by both the fiscal balance and the interest rate. Analyzing the behavior of both instruments and both targets together leads to unexpected conclusion that the case for countercyclical fiscal policy may become stronger, not weaker, as the debt-to-GDP ratio rises.

Our analysis starts from Tinbergen's familiar language of policy targets and instruments. (Tinbergen, 1952). We differ from previous work in presenting a simple framework within which the joint effects of the two policy instruments on the two targets can be analyzed. This consists of a version of the "three-equation" model familiar from macroeconomics textbooks, plus the law of motion of government debt. In the second part of the paper, we examine the implications of alternative policy rules in this framework. In the brief third section, we explore how this analysis applies to concrete historical data for the postwar United States.

From the formal analysis, we draw two conclusions. First, we show that there will be one set of combinations of interest rates and fiscal balances that will keep output at potential, and another set that will hold the debt ratio constant. A unique point in fiscal balance-interest rate space is in both sets and satisfies both conditions. This can be represented as a pair of loci in interest rate-fiscal balance space, with the unique combination of interest rate and fiscal balance consistent with both debt stability and a zero output gap found at the intersection of the two loci. The location of this point does not depend on which instrument is assigned to which target. This has a surprising implication: The familiar instrument assignment in which the interest rate is set by the monetary authority to keep output at potential, and the fiscal balance is set to hold the debt-GDP ratio constant, will in general imply the exact same values for the interest rate and fiscal balance, as a rule in which the fiscal balance is set to keep output constant, and the monetary authority sets the interest rate at the level required to hold the debt-GDP ratio constant. In general, macroeconomic outcomes when the monetary authorities are responsible for output and the fiscal authorities face a binding budget constraint, will be indistinguishable from outcomes when the fiscal authorities ignore the debt ratio and focus only on output.

Our second conclusion points in a different direction. While the two instrument assignments imply the same equilibrium values for the two instruments, they do not imply the same behavior away from equilibrium. So they may have different stability properties. We find that, for realistic parameter values, the "functional finance" assignment in which the fiscal balance targets output and the interest rate targets the debt ratio, always converges; but the orthodox "sound finance" assignment converges only if the initial debt ratio is not too high. This is because the higher the debt ratio, the more changes in the debt ratio depend on the effective interest rate, as opposed to the current fiscal balance. Thus, from our point of view, the familiar metaphor of "fiscal space" is exactly backward. In fact, the higher is the current debt ratio, the stronger is the argument for countercyclical fiscal policy, because at high debt ratios the interest rate instrument will be required to stabilize the debt ratio. This is consistent with the historical experience that when public debt ratios are sufficiently high, moderating debt service costs for the government becomes the primary consideration for central bank rate-setting.

In the third section, we explore whether the adjustment dynamics discussed in the first section could be relevant to concrete developments in the United States. In particular, is it plausible that output fluctuations could be, at least in part, the result of instability endogenously generated by interactions between the two policy instruments? We tentatively suggest that much of recent macroeconomic history can be understood as a "sound finance spiral" of the sort described in the first section.

The analysis here bears a family resemblance to the literature on what has been termed the "current consensus assignment," in which monetary policy is preferred for demand management while fiscal policy is assigned to debt stabilization.(Kirsanova, Leith and Wren-Lewis, 2009) Representative papers exploring the conditions under which the consensus assignment dominates or does not include Blake, Vines and Weale (1988), Eser (2006), and Kirsanova and Wren-Lewis (2012) among others. Unlike most the papers in this literature, we do not base our arguments on any account of intertemporal optimization, but work only with a few reduced-form aggregate relationships. Somewhat similar conclusions are reached through an explicit intertemporal optimization framework by Woodford (2001), as discussed in the third section of the paper. Nonetheless, we believe there is value in demonstrating that a simple aggregate model, of the kind used in policy settings and for forecasting as well as in classroom settings, has different implications for macroeconomic policy than is widely believed.

1 A Simple Framework for Macroeconomic Policy Analysis

1.1 The Consensus Macroeconomic Model

Our starting point is the simple model of aggregate behavior that that underlies most contemporary discussions of macroeconomic policy.[1] The model embodies four key assumptions, none controversial.

1. First, the interest rate is set by the monetary authority.

[1] For critical discussions of the "new consensus" macroeconomic models that we follow here, see Palacio-Vera (2005) and Carlin and Soskice (2009). For examples of major macroeconomic forecasting models fundamentally based on the textbook three-equation model, see Brayton and Tinsley (1996) and Hervé et al. (2011).

The claim that it is both necessary and possible for the monetary authorities to maintain the prevailing interest rate at a level consistent with price stability has been a central tenet of macroeconomic policy at least since it was formulated by Wicksell early in the last century. (Wicksell, 1936) Despite the central role of this claim in modern macroeconomics, it is not entirely clear how the monetary authority is able to set the terms of credit transactions throughout the economy; it is sometimes suggested that its apparent ability to do so historically may have depended on institutional conditions that no longer hold, or may cease to hold in the future. (Friedman, 1999) These concerns are reinforced by the empirical fact that real economies have many different interest rates, which do not always move together. Nonetheless, macroeconomic policy discussions are normally conducted in terms of "the" interest rate. In the equations below, we use i to refer to the average inflation-adjusted rate on outstanding government debt. But our model naturally generalizes to the case where the various rates do not move together. How, or whether, the monetary authority is able to set the prevailing rate of interest is an important question, but not one that it is necessary to pursue here, since all modern macroeconomic models begin with the assumption that the interest rate is fixed by the monetary authority. For a fuller discussion of this issue, see Woodford and Walsh (2005, p. 30-45).

2. Second, inflation is a positive function of the current level of output, along with its own past or expected values and other variables. Fiscal and monetary policy affect inflation only via output. This assumption is formalized as a Phillips curve:

$$\hat{P} = \hat{P}(Y - Y^*, \hat{P}^E), \tag{1}$$

$$\hat{P}_{Y-Y^*} > 0$$

$\hat{P}$ is the inflation rate, $\hat{P}^E$ is the expected inflation rate, Y is output as measured by GDP or a similar variable, and Y^* is potential output. For our purposes it does not matter how inflation expectations are formed. In modern macroeconomic models, it is normally assumed that there can be no persistent deviations of expected from realized inflation, so that the long-run Phillips curve is vertical, with $Y = Y^*$ the unique level of output at which inflation is stable. Some heterodox economists continue to argue that output and inflation should be treated as distinct policy targets even in the long run. (Michl, 2008) But a vertical Phillips curve is not required to treat inflation and output as a single target; it is sufficient that the long-run curve be steep, and/or that there is a well-defined tradeoff between the two targets in the implicit social welfare function maximized by policy. (Taylor, 1998)

3. Third, output is a negative function of the interest rate, and a negative function of the fiscal balance (or positive function of the government deficit) via the multiplier.[2]

   This assumption is formalized as an IS curve:

$$Y = A - \eta i Y^* - \gamma b Y + \tau i d Y \tag{2}$$

   A is autonomous spending, here defined as the level of output when both the interest rate and fiscal balance are zero. i is the average interest rate on government debt. For now, we consider i to be the "real" (that is, inflation-adjusted) interest rate. η is the semi-elasticity of output with respect to the interest rate, that is, the percentage increase in output resulting from a point reduction in the interest rate. In the interests of mathematical simplicity, the effect of interest rates on real activity is expressed in terms of potential output Y^* rather than current output Y. Note that the value of η reflects both the responsiveness of real activity to changes in interest rates, and the strength of the correlation of the marginal rate facing private borrowers with the average rate on public debt. So no special assumption is needed about whether all interest rates move one for one with the policy rate. If we think they respond less than proportionately, we simply use a lower value of η. b is the primary balance of the government, with positive values indicating a primary surplus and negative values a primary deficit. γ is the multiplier on whatever mix of tax and spending changes are used to adjust the government fiscal balance. (A multiplier of zero (Ricardian equivalence or full crowding out) is included as a special case of $\gamma = 0$.) d is the ratio of government debt to current output. τ is the multiplier on interest payments; its helpful to allow the possibility that this multiplier is different from the one on the changes in tax and spending captured by changes in d.

4. Our fourth assumption is simply that the end of period debt is equal to the start of period debt plus the accumulated primary deficits and interest payments. This gives us the law of motion of government debt, "the least controversial equation in macroeconomics." (Hall and Sargent, 2011)

$$\Delta d = \frac{i - g}{1 + g} d - b \tag{3}$$

[2] Modern macroeconomic models derive the path of output from a Euler equation, which is intended to capture a process of intertemporal optimization. However, in policy applications this equation is invariably linearized into a form similar to Equation 2. (Billi, 2012)

where i, d and b are defined as above and g is the growth rate of output, again net of inflation. Equation 5 is not an accounting identity since it will be violated not only in the case of defaults but also by sales of public assets, government assumptions of private debts, and other transactions that affect the public debt but are not included in standard measures of the fiscal surplus or deficit. In some cases, such as Ireland in 2009-2011, such transactions may dominate the evolution of the public debt. This possibility complicates the question of what constitutes a stable debt trajectory, but these complexities are beyond the scope of this paper. Here, we assume that Equation 5 holds exactly.

These four standard assumptions are all that is required of the analysis that follows. In the formal analysis we can assume that all nominal variables have been appropriately adjusted for inflation, so that we are working with "real" variables. The need to adjust nominal interest rates for inflation is not in general a point of contention, even among perspectives that diverge sharply in other respects. (Smithin, 2006) But it is important to keep in mind that in empirical work and in practical policymaking, the appropriate form of inflation adjustment is seldom obvious – neither the choice of price index nor of the period inflation over which should be subtracted from a given nominal interest rate observation, is straightforward. (Knibbe, 2015)

1.2 Targets

Any version of the Phillips curve is sufficient to make output and inflation a single target, for the purposes of stabilization policy. So for simplicity, we assume that policy targets an output gap of zero, that is, $Y = Y^*$.

We will now work in terms of the output gap y and replace autonomous expenditure with z, where

$$y = \frac{Y - Y^*}{Y^*}$$

$$z = \frac{A - Y^*}{Y^*}$$

In other words, z is the output gap when the primary deficit and interest rate are both zero, measured as a fraction of potential output.

We now rewrite the IS relationship as

$$y = z - \eta i - \gamma b + \tau i d$$

Then for $y = 0$, we need:

$$i = \frac{z - \gamma b}{\tau d - \eta} \tag{4}$$

Equation 4 simply means that maintaining output at potential requires the interest rate (i) to fall when autonomous expenditure (z) falls or when the primary surplus (b) rises . The degree to which i must fall depends on the relative responsiveness of output to the fiscal balance and to the interest rate, and on how much consumption depends on income from government bond holdings (τd). With a high η , i needs to fall less to compensate for a rising primary surplus; with a high γ, i needs to fall more.

This gives us our price stability locus. Next we consider debt sustainability.

The law of motion of government debt is:

$$\Delta d = \frac{i - g}{1 + g} d - b \tag{5}$$

where d is the current debt-GDP ratio, i is the effective interest rate on outstanding government debt, g is the growth rate, and b is the primary balance, with positive values for surpluses. Then to hold d constant, we need:

$$i = \frac{dg - b(1 + g)}{d} = g + \frac{1 + g}{d} b \tag{6}$$

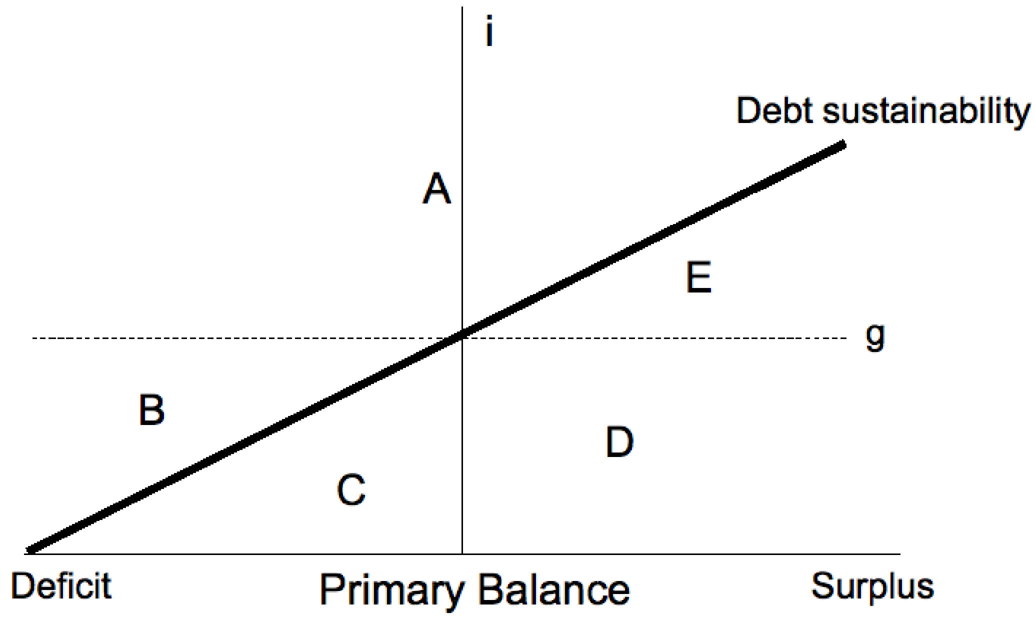
This is the constant debt ratio locus.

Again, the interpretation is straightforward. For a given debt-GDP ratio, an increase in the growth rate of GDP (g) or the primary surplus(b) will reduce the debt to GDP ratio unless counteracted by an increase in the interest rate to maintain the current ratio.

There is not a consensus on the meaning of debt sustainability. [3]The weakest form allows the debt-GDP ratio converge to any finite value. The next strongest is that the ratio remain at or below its current level. The strongest version requires the ratio to remain at or below some exogenously given level. The latter two conditions may be framed as equalities or inequalities; a budget position that implies that the debt fall to zero, or that the government ends up with a positive asset position,

[3]See the discussion of alternative debt-sustainability targets in Aspromourgos, Rees and White (2010) and Pasinetti (1998) Portes and Wren-Lewis (2014) expresses the common view that optimal fiscal policy implies that the debt ratio follows a random walk; this is equivalent to our debt-sustainability condition that the authorities target the current debt ratio, whatever it may be.

Figure 1: Alternative Debt Sustainability Conditions



The line labeled debt sustainability indicates those combinations of interest rates and fiscal balances for which the debt to GDP ratio is constant. It passes through the vertical axis at $i = g$ and has a slope of $\frac{1+g}{d}$. In area A, above the locus with $i > g$, the debt-GDP ratio rises to infinity. In area B, above the locus but with $i < g$, the debt-GDP ratio rises toward some finite value. In area C, below the locus with a primary deficit, the debt-GDP ratio falls to some finite value. In area D, below the locus with a primary surplus and with $i < g$, the debt-GDP ratio falls to zero and the government then acquires a positive net asset position which rises to some finite fraction of GDP. Finally, in area E, below the locus with a primary surplus and $i > g$, the debt-GDP ratio falls to zero and the government then acquires a positive net asset position which rises without limit as a fraction of GDP.

may or may not be considered sustainable. In the absence of any strong reason for preferring one or the other, we use the middle condition, that the debt ratio remain constant at its current level. Our results could be easily be extended to the third, strongest case. But we have chosen not to do so here, since this would involve adding one or more additional parameters for only a small gain in generality.

Alternative definitions of debt sustainability are shown in Figure 1. Only area A does not satisfy any definition of debt sustainability.

If we define debt sustainability as the debt ratio being stable at its current level, then Equation 6 is the condition for debt sustainability. If we define debt sustainability as a constant or falling debt ratio, then we can write:

$$i \leq g + \frac{1+g}{d} b$$

If we define debt sustainability as the condition that the debt ratio not to rise without limit, then it's sufficient to meet either the above condition or $i < g$.

Combining our price stability locus (Equation 4) and constant debt ratio locus (Equation 6) gives us the unique values for i and b for which output is at potential and the debt-GDP ratio is constant.

$$i = \frac{z(1+g) + \gamma g d}{\gamma d + (\eta - \tau d)(1+g))} \approx \frac{z}{\eta + (\gamma - \tau) d} \tag{7}$$

$$b = \frac{zd - gd(\eta - \tau d)}{\gamma d + (\eta - \tau d)(1+g)} \approx \frac{z}{(\gamma - \tau) + \frac{\eta}{d}} \tag{8}$$

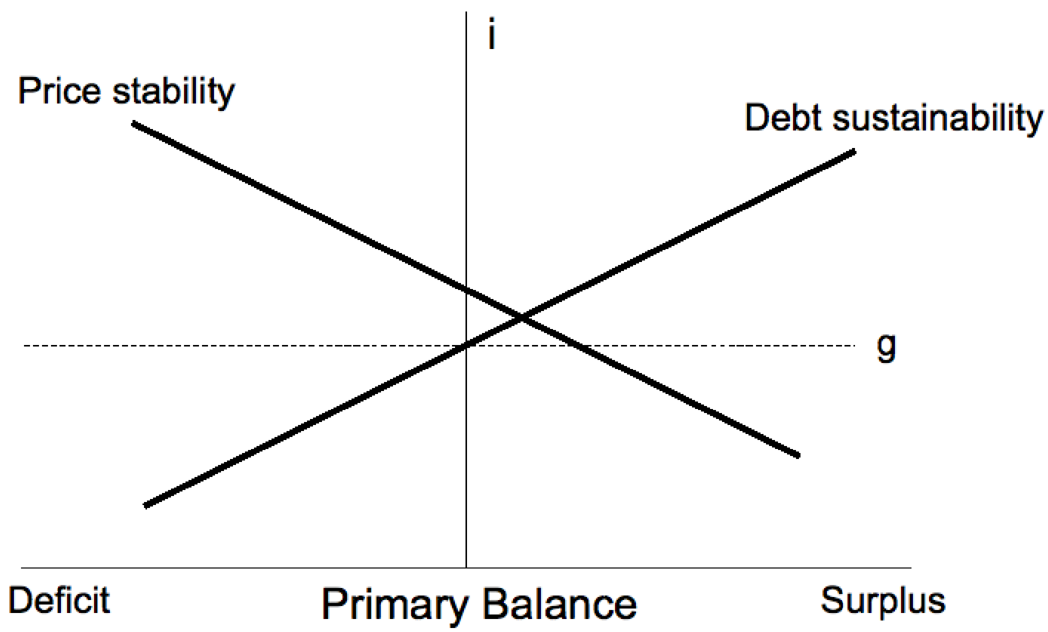
Both approximations are derived from the assumption that g will always be much smaller than one.

The equations each have a natural interpretation. The value for i indicates that for a given τ and γ, when debt (d) is low, the equilibrium interest rate depends mostly on autonomous expenditure. The dependence of the equilibrium interest rate to autonomous expenditure also depends on η, with a greater dependence when η is lower (i.e. the interest elasticity of output is lower). With high levels of d, the equilibrium interest rate depends less on autonomous expenditure, unless the fiscal multiplier is close to zero. The equilibrium value of b indicates that as d rises, b must approach $Z/(\gamma - \tau)$. In other word, as the debt ratio rises, the equilibrium primary surplus depends more on autonomous expenditure, and less on the debt ratio. The two equations together telegraph a finding that we discuss in greater detail later. Simply put, as the debt ratio rises, the role of i in maintaining potential output must diminish while that of the budget balance b increases.

We can represent the two loci graphically with the interest rate on the vertical axis and the primary balance on the horizontal axis, as shown in Figure 2. The constant debt ratio locus slopes downward, and passes through the point ($d = 0$, $i = g$). The slope of the locus depends on the current debt ratio: It is vertical through $b = 0$ when the debt ratio is zero, and approaches a horizontal line at $i = g$ as the debt ratio rises to infinity. (This expresses graphically the same point made above, that debt stability depends mostly on the fiscal balance when the debt ratio is low, and increasingly on the interest rate as the debt ratio grows higher.) If τ is zero, the price stability locus must slope upward. If τd is large, it may slope downward instead. (In this case the sound finance policy rule will move the economy away from potential output.) If $\eta - \tau d = 0$ – changes in interest rates do not affect output – then the price stability locus will be a vertical line at some value of d. Conversely, if $\gamma = 0$ – that is, full crowding out or Ricardian equivalence – the price stability locus will be horizontal at $i = \frac{1}{\eta}Z$.

A change in autonomous demand A – which captures any exogenous change in demand that policy must respond to – shifts the price stability locus horizontally by an amount equal to A. In any period in which the economy is not on the constant debt ratio locus, there is a change in d. An increase in d rotates the constant debt ratio locus clockwise around the point ($d = 0$, $i = g$). With $\tau > 0$, then an increase in d also shifts the price stability locus downward and rotates it clockwise, eventually through the vertical (when $\tau d = \eta$) and to a horizontal line at $i = 0$.

Figure 2: Price Stability and Debt Sustainability Conditions



The line labeled price stability indicates those combinations of interest rates and fiscal balances for which $Y = Y^*$. It passes through the vertical axis at $i = \frac{1}{\eta}Z$ and has a slope of $\frac{\gamma}{\eta}$. The line label debt sustainability indicates those combinations of interest rates and fiscal balances for which the debt to GDP ratio is constant. It passes through the vertical axis at $i = g$ and has a slope of $\frac{1+g}{d}$. The "sound finance" instrument assignment implies that interest rates are adjusted towed the price stability locus and the fiscal balance is adjusted toward the debt sustainability locus; out of equilibrium, this implies movement in a clockwise direction. The "functional finance" instrument assignment implies that interest rates are adjusted toward the debt sustainability locus and the fiscal balance is adjusted toward the price stability locus; out of equilibrium, this implies movement in a counterclockwise direction.

2 Alternative Policy Rules

2.1 Sound Finance and Functional Finance

A useful way of thinking about policy in this framework is in terms of two alternative instrument assignments, which we call "sound finance" and "functional finance." Sound finance sets i^* as whatever value of i satisfies Equation 2 at last period's (or expected) b, and b^* at whatever level of b satisfies Equation 5 at last period's (or expected) i. In other words, the interest rate instrument is assigned to the output target, and the budget balance is assigned to the debt ratio target. Functional finance sets b^* as whatever value of b satisfies Equation 2 at last period's (or expected) i, and i^* at whatever level of i satisfies Equation 5 at last period's (or expected) b. In other words, the the budget balance instrument is assigned to the output target and the interest rate instrument is assigned to the debt ratio target.[4]

Equations 7 and 8 describe the unique equilibrium combination of i and b for which the output gap is zero and the debt-GDP ratio is constant. Thus, abstracting from any changes in the debt ratio that occur during the process of convergence, if the policy rules converge at all they will bring the economy to the same final state regardless of which set of policy rules is being followed. In other words: Suppose the budge authority is following some fiscal rule that satisfies the conditions for debt sustainability, whatever they may be. And suppose that, given that fiscal rule, the monetary authority is able to follow an interest rate rule that keeps output at its target level. Then it must also be possible for the fiscal authority to instead ignore the debt ratio and set the budget balance at whatever level leads to the target level of output, and the monetary authority to then set the interest rate at whatever level stabilizes the debt ratio. This equivalence between the superficially contrasting "sound finance" and "functional finance" policy rules is the first significant result of our analysis.

If the debt-targeting instrument adjusts much faster than the output-targeting instrument, then the economy, if it converges at all, will arrive at the point which satisfies Equations 7 and 8 for initial debt d_0, regardless of the initial interest rate and budget balance. Otherwise, the debt ratio will change over the course of the adjustment process, and the final state will in general depend on which set of rules are being followed, as well as on the initial state of the economy and the values of the adjustment speed parameters. The outcome in this case cannot be derived analytically but must be simulated numerically.

[4] We take the terms "sound finance" and "functional finance" from Lerner (1943). But it's important to note that, unlike us, Lerner did not treat the debt ratio as a target for policy, but rather assumed that it would passively adjust to accommodate fiscal policy and the independently determined interest rate. We thank Peter Skott for clarifying this point.

Next, we explore convergence conditions under the two assignments.

2.2 Convergence Under Alternative Instrument Assignments

We are here interested in the behavior of the two rules in achieving convergence to equilibrium from different starting points in the (b,i) space.

The IS curve from the previous sections is now given by

$$y = z - \gamma b - \eta i + \tau d i \tag{9}$$

The linearized equation of motion of the debt-to-income ratio is

$$\dot{d} = -b + (i - g)d \tag{10}$$

Our targets therefore are $y = 0$ and $\dot{d} = 0$.

In a sound finance regime, interest rates fall when output rises above the level consistent with full employment and price stability, and rise when output falls below this level.[5] The fiscal balance is adjusted toward surplus when the debt ratio is rising and allowed to move toward deficit when the debt ratio is falling. The equations of adjustment are therefore given by :

$$\dot{i} = \alpha[\frac{1}{\eta}(z - \gamma b + \tau d i) - i] \tag{11}$$

$$\dot{b} = \beta[-(i - g)d - b] \tag{12}$$

where α and β are adjustment speed parameters.

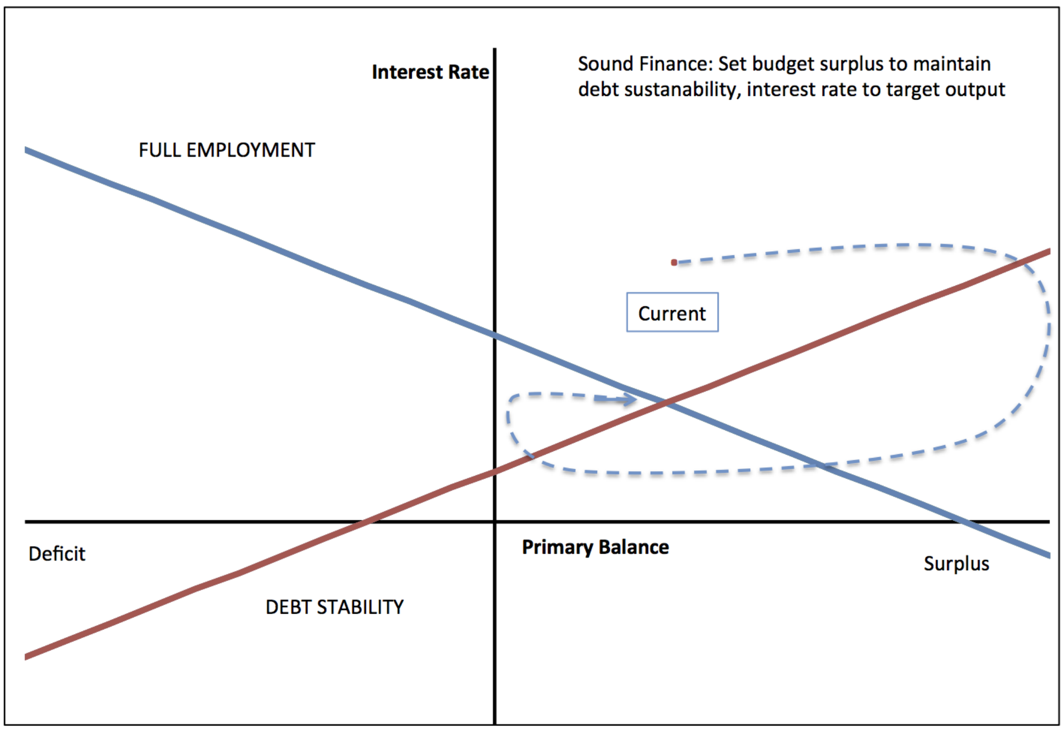
The interpretation of α and β requires some explanation. In reality, the speed with which fiscal and monetary policy respond to macroeconomic variables depends on a whole range of political and institutional factors particular to the country and time period. So we do not want to incorporate any strong assumptions about the policy adjustment process. What α and β reflect is simply how fast the fiscal balance and interest rate are generally adjusted in a particular context, relative to the frequency with which the authorities receive news about the target variable. This parameter therefore is purely descriptive, and will have some value for *any* policy adjustment process. This value will range from one when the policy instrument

[5] Equation 11 is equivalent to a Taylor Rule.

moves instantaneously in response to a change in the target variable, to zero when the instrument does not respond to changes in the target.

To illustrate the process of adjustment let us assume that we are at a point of rising debt (i.e above the debt stability locus), but also below potential output. The sound finance implies moving vertically toward potential output locus and horizontally toward debt stability locus as depicted in figure 3. In the figure shown, the budget is moved towards surplus in order to achieve debt sustainability, while the interest rate is set to target output . As drawn, despite overshooting initially, the system spirals clockwise inwards towards the equilibrium.

Figure 3: Convergence using Sound Finance Rule



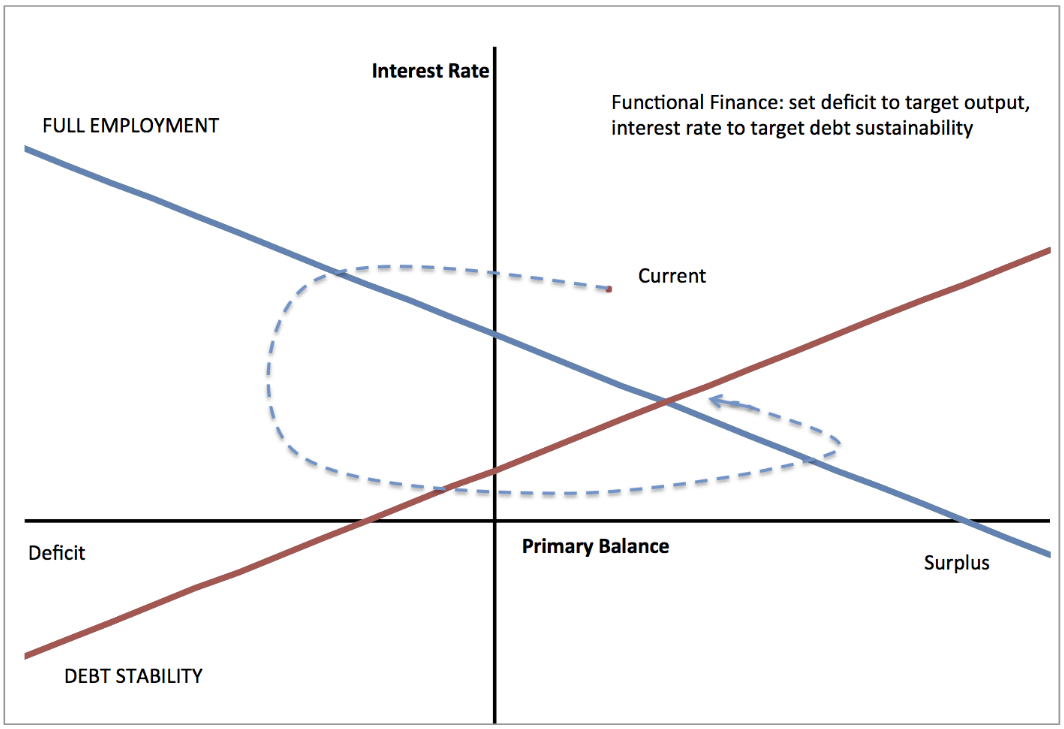
In a functional finance regime, the fiscal balance moves toward surplus when output rises above the level consistent with full employment and price stability, and toward deficit when output falls below this level. (The defining feature of functional finance is that the fiscal balance is *not* responsive to the debt ratio.) The interest rate is adjusted to keep the debt ratio table, so it is reduced when the debt ratio is rising. The rules can therefore be written as:

$$\dot{i} = \alpha[g + \frac{b}{d} - i] \tag{13}$$

$$\dot{b} = \beta[\frac{1}{\gamma}(z - \eta i + \tau di) - b] \tag{14}$$

This can be depicted in Figure 4. Starting from the same position as before, the budget balance is moved to deficit in order to hit full employment while the interest rate is lowered to achieve debt sustainability. This is drawn as a counterclockwise spiral inwards towards the equilibrium.

Figure 4: Convergence using Functional Finance Rule



While we have drawn the spirals converging, whether in fact the system converges depends on the parameters and on the initial values of the debt ratio and output gap.

Given a set of linear differential equations as we have with both rules, for stability of the equilibrium we need (from the Routh-Hurwitz conditions) CITE that the Jacobian Matrix satisfies the following :

A. $tr(J) < 0$

B. $det(J) > 0$

2.2.1 Convergence Conditions for the Sound Finance Rule

For the sound finance rule, the Jacobian Matrix is given by

$$J_{sf} = \begin{bmatrix} -\alpha(1-\frac{\tau d}{\eta}) & -\alpha\frac{\gamma}{\eta} \\ -\beta d & -\beta \end{bmatrix}$$

This gives us

$$tr(J_{sf}) = -(\alpha(1-\frac{\tau d}{\eta}) + \beta)$$

$$det(J_{sf}) = \alpha\beta(1-\frac{\tau d}{\eta} - \frac{\gamma d}{\eta}) = \alpha\beta(1-\frac{d}{\eta}(\tau+\gamma))$$

Condition A requires that $\alpha\beta(1-\frac{d}{\eta}(\tau+\gamma)) > 0$ this can be rearranged to be

$$d < (1+\frac{\beta}{\alpha})\frac{\eta}{\tau} \tag{15}$$

Condition B requires that

$$\alpha\beta(1-\frac{d}{\eta}(\tau+\gamma)) > 0$$

which can be rewritten as

$$d < \frac{\eta}{\tau}\frac{1}{(1+\frac{\gamma}{\tau})} \tag{16}$$

It is easy to see that equation 16 is binding since the maximum threshold for d implied by equation 15 is always larger than the maximum threshold level of d for Equation 16

We can summarize the implications of Equation 16 as follows:

The sound finance rule will only converge below some critical value of the debt ratio. That critical value will depend on the three parameters. A low threshold – and hence a greater probability of divergence under the sound finance rule – will result if η is small relative to τ and both are both small relative to γ. Thus for example, with plausible values of the parameters ($\eta = 1$, $\tau = .1$ $\gamma = 1.5$), the threshold level of d is 0.63. Below this debt ratio, a small departure of the instruments from their equilibrium values will be diminish over time. Above this ratio, a small departure of the instruments from their equilibrium values will result in explosively larger adjustments away from equilibrium, In general, then, stability under sound finance requires that the direct effect of interest rates on expenditure be relatively strong compared to effects of income changes due to either fiscal balance or interest payments. Note that the critical question is the relationship of γ and η; τ, which will certainly be less than γ, is less important. With $\tau = \gamma$, the convergence threshold will be $d = \frac{\eta}{2\gamma}$. As τ goes to zero, the convergence criteria will approach $d = \frac{\eta}{\gamma}$.

Note that we have ignored here changes in the debt ratio and the output gap resulting from movements of the instruments out of equilibrium. Incorporating these effects would only strengthen the conclusion that the functional finance rule converges only at low debt ratios. We return to this point in the Conclusion.

2.2.2 Convergence conditions for the Functional Finance Rule

For the functional finance instrument assignment, the Jacobian Matrix is given by

$$J_{ff} = \begin{bmatrix} -\alpha & \frac{\alpha}{d} \\ -\beta\frac{\eta-\tau d}{\gamma} & -\beta \end{bmatrix}$$

This gives us

$$tr(J_{ff}) = -(\alpha + \beta)$$

$$det(J_{ff}) = \alpha\beta(1 - \frac{\tau d - \eta}{\gamma d})$$

Condition A is always satisfied.

Condition B requires that :

$$\alpha\beta(1 - \frac{\tau d - \eta}{\gamma d}) > 0 \tag{17}$$

This is always satisfied for: $\gamma > \tau$. If $\tau > \gamma$, the condition requires that

$$d < \frac{\eta}{\tau - \gamma}$$

Thus, from equation 17 all that is required is for the multiplier to be larger than the effect of income from interest receipts. For virtually all historically based parameters, this will be the case. Even in the highly implausible case of $\tau > \gamma$ (i.e.

private expenditure more sensitive to interest payments than to the baseline mix of spending and tax changes), the difference must be large for instability to arise.

Combining Equations 15 through 17, we draw a general conclusion about the stability properties of the two rules. Both rules are stable for a range of debt values. Beyond a certain value of debt, only functional finance will remain a viable assignment for convergence to equilibrium.

Specifically, for

$$\frac{\eta}{\tau+\gamma} < d < \frac{\eta}{\tau-\gamma}$$

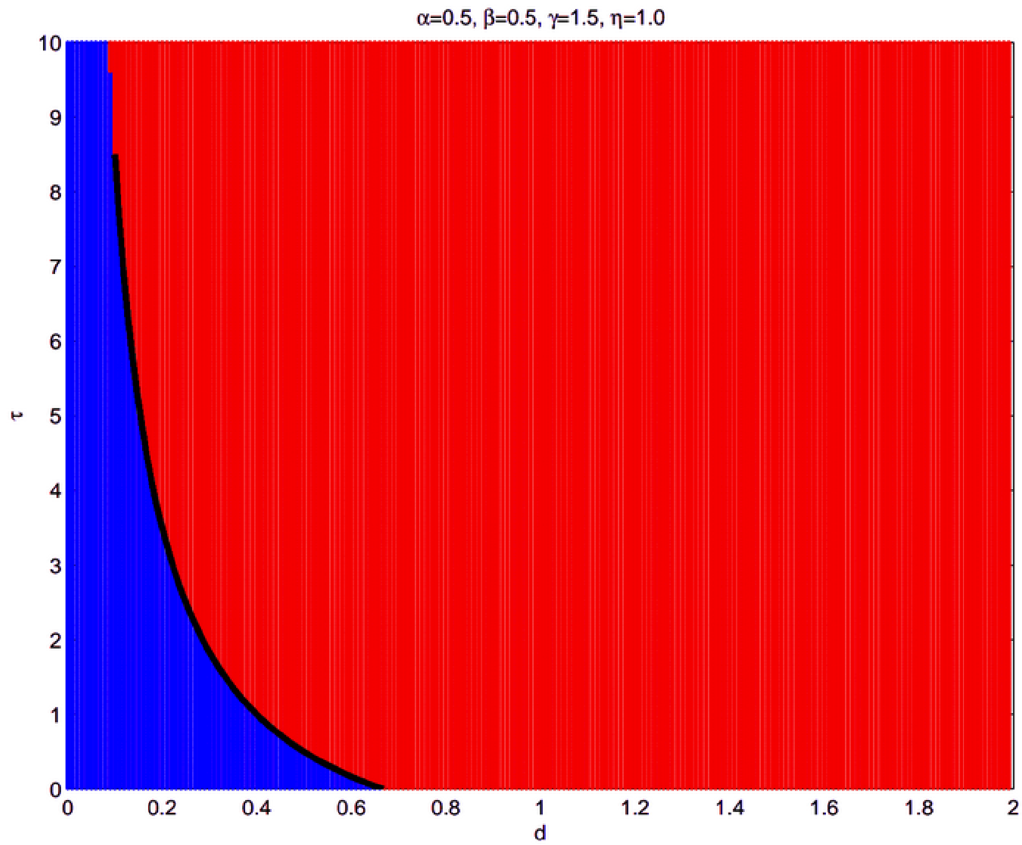
only the functional finance instrument will converge.This can be seen visually in Figure 5 and Figure 6. There, the red areas reflect combinations of τ and d for which the sound finance and functional finance assignment leads to instability, while conversely the blue areas reflect combinations that will lead to stability for a given set of plausible parameters ($\alpha, \beta = 0.5$, γ=1.5, η=1.0). As the figure shows, the functional finance instrument is the only assignment that leads to stability after a leverage ratio of 0.6. Moreover, it is stable even for large values of τ.

Note that these conclusions have implications beyond the exact convergence conditions. They imply that even where both rules converge, convergence will be relatively faster under the sound finance rule when the debt ratio is low, and relatively faster under the functional finance rule when the debt ratio is higher. So the qualitative conclusion that the functional finance rule becomes relatively more consistent with macroeconomic stability, and the sound finance rule less so, as the debt ratio rises, does not depend on whether the convergence criteria are satisfied in any particular case.

Based on this analysis, we can see that when the debt-GDP ratio is sufficiently high, stability *requires* that the interest rate instrument target (mainly) the stability of the debt ratio, and the fiscal balance target (mainly) the output gap. Thus, under a very general set of assumptions, the common metaphor of "fiscal space" gets the relationships between debt levels and policy backward. Stability requires that the fiscal authorities make less effort, rather than greater effort, to stabilize the public debt as the debt to GDP ratio rises. Countercyclical fiscal policy not only remains possible at high debt levels, but becomes obligatory.

While the results are clear, the intuition behind them may not be immediately obvious. So it is worth thinking through why instability arises. In effect, the sound finance assignment suggests that the budget authority should respond to signals from the debt path to decide on spending and tax levels. A rising debt-GDP ratio is a signal that spending is excessively high and/or taxes are excessively low, and a falling debt-GDP ratio is a signal that spending is needlessly low and/or taxes are needlessly high. interest rates as well as tax and spending decisions influence

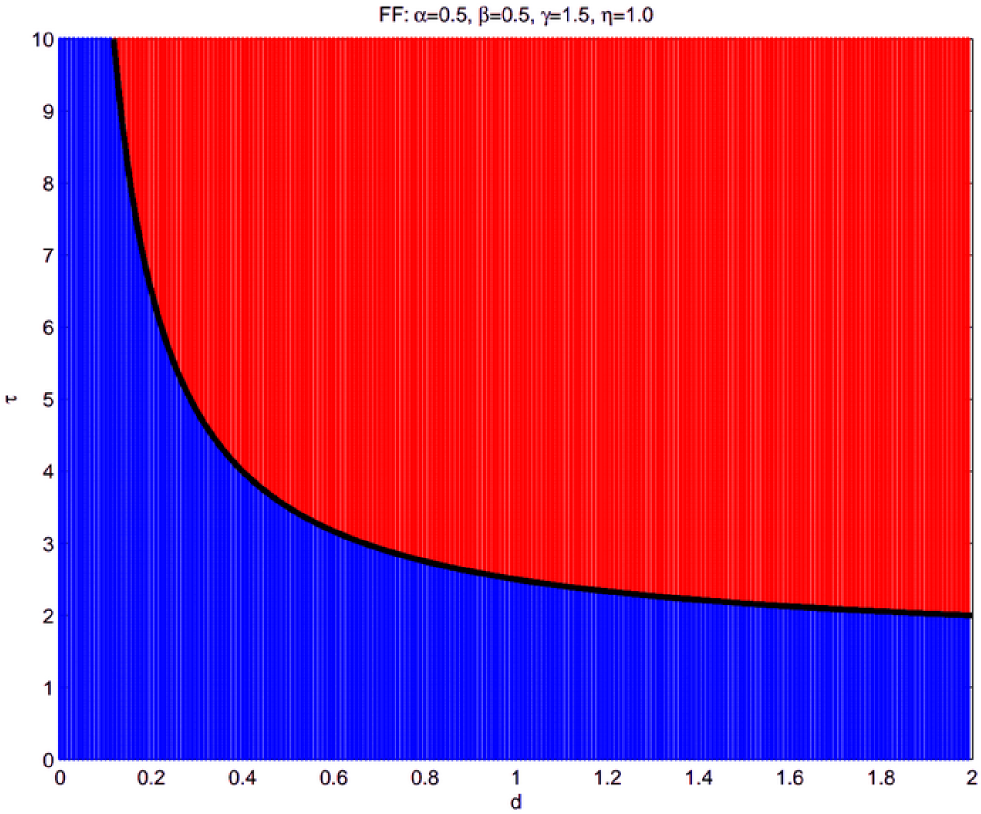
Figure 5: Regions of Stability and Instability using the Sound Finance assignment



the debt path. If the monetary authorities do not take into account the signals changes in policy send to the budget authorities, then changes in monetary policy will induce additional, unintended changes in the fiscal balance that will amplify the initial effect on output. The larger the current debt, the larger these unintended effects will be, since the bigger an impact a change in interest rates will have on the budget position. These unintended effects will also be larger if the interest rate set by the monetary authority has a stronger relationship with public than with private borrowing costs. Changes in the fiscal position carried out to stabilize the debt ratio will, in turn, affect demand and induce further interest rate changes. When the cross-effects are large, this will lead to a situation where each adjustment in one instrument induces a larger adjustments in the other.

It's important to stress that this conclusion does not depend on interest payments having any effect on output. They require only the mathematical fact that as the higher the level of existing debt, the more changes in debt depend on interest rates

Figure 6: Regions of Stability and Instability using the Functional Finance assignment



and the less they depend on the primary balance. Instability is possible only under the sound finance rule because while the effects of the two instruments on output are stable, the effect of fiscal policy on the debt ratio goes to zero as the debt ratio rises.[6] This requires ever-larger adjustments of the fiscal balance in response to interest rate changes by the monetary authority.

One may ask whether, even if this analysis is formally correct, it offers a useful tool for understanding the evolution of macroeconomic targets and instruments in real economies. In the final section of the paper we address this question, using the framework developed here to analyze the trajectory US macroeconomic policy over recent decades.

[6]Strictly speaking, instability is also possible where the monetary authority is trying to stabilize the debt ratio at a very low level, but this case is irrelevant since in the real world when public debt is just a few percent of GDP it is not an important target of policy.

3 Historical Applications

3.1 A Role for Endogenous Policy Cycles in Recent Macroeconomic History?

Analysis of macroeconomic policy typically focuses on optimal policy rules. The concrete conduct of policy is less often an object of analysis.[7] But a natural extension of the analysis in the previous sections is to ask, has the interaction between policy instruments played a part in macroeconomic instability historically?

The logic is straightforward. Under the policy orthodoxy of the postwar period, the policy interest rate (or equivalent monetary instrument) has been used to target output, while the federal budget position has been adjusted to target debt stability. These rules were, of course, suspended during World War II, and were contested to some extent into the 1970s. But since 1980 or so, this "sound finance" instrument assignment has been held to fairly strictly. Indeed, the delegation of responsibility for output stabilization exclusively to the central bank has been seen as a major step forward in macroeconomic policy, a "glorious counterrevolution" that was "directly responsible ... for the virtual disappearance of the business cycle." (Romer, 2007)

Under this assignment, we would expect to see the policy instruments follow clockwise cycles as in Figure 3. An increase in the interest rate will tend to increase the debt ratio at a given primary balance, requiring the budget authorities to shift the primary balance toward surplus. A surplus will tend to reduce demand, leading the monetary authorities to reduce interest rates. Lower interest rates will imply slower growth of the debt ratio, allowing the fiscal authorities to shift the budget back toward deficit. And so on. The amplitude and frequency of these cycles will depend on the speed with which aggregate expenditure responds to the policy variables, the speed with which the effective interest rate facing the government responds to the policy interest rate, and the speed with which each instrument responds to deviations in its target, as well as on exogenous shifts in aggregate expenditure. In Section 2.2, we suggested that for plausible parameter values, these cycles may even amplify rather than dampen over time – that is, there may not be convergence, especially under the sound finance rule when the debt-GDP ratio is already high. Even if policy cycles are dampened, they still represent an independent source of macroeconomic instability, since any initial shift in demand will produce "echoes" as the policy variables spiral back toward equilibrium. In principle, some large fraction

[7] The obvious exception is the public choice literature, and the related idea of time-inconsistency of policy, as well as the broader but less explicitly theorized presumption that macroeconomic policy in democratic polities suffers from a bias toward deficits and inflation. (Portes and Wren-Lewis, 2014). For a critical assessment of time-inconsistency arguments about macroeconomic policy, see Bibow (2004).

of business cycles could be explained in terms of endogenous interaction between policy instruments, rather than exogenous "shocks".

The view that business cycles are largely produced by stabilization policy is most commonly associated with Milton Friedman and more recent monetarists. The monetarist story involves only a single policy instrument, with cycles being the result of lags in both the implementation and effects of policy changes. (Friedman, 1960) An account of destabilizing interaction between monetary and fiscal policy closer in spirit to the one proposed here, is found in Woodford (2001). Woodford argues there that the question of what monetary policy rule is the best route to price stabilization cannot be separated from what fiscal rule is followed by the budget authorities. Similarly, any target for the public debt cannot be reduced to a budget rule, but depends on the policy followed by the monetary authorities. [8] Woodford considers the ways in which a failure to take this interdependence into account, can lead to destabilizing interactions between the policy instruments.We suggest that this is more than a theoretical possibility. In particular, we suggest that the evolution of output and the federal budget position over the last 40 years can be understood as a long "policy cycle" of the kind analyzed in Section 2.2. The narrative we suggest is the following:

In the immediate postwar period, the United States was effectively operating under a functional finance instrument assignment, with interest rates set to stabilize the federal debt and fiscal policy playing the central role in keeping output at potential. Over the next 25 years, the assignment of instruments was gradually switched, with interest rates moving to target mainly output in the 1950s, and fiscal policy coming to target government debt by the end of the 1970s. (Sylla, 1988) At this time, the

[8] As Woodford observes, this interdependence between the policy instruments is rejected by today's macroeconomic orthodoxy: "It is now widely accepted that the choice of monetary policy to achieve a target path of inflation can ..., and ought, to be separated from ... the choice of fiscal policy." Most macroeconomists think that monetary policy is irrelevant for the debt-GDP ratio, he says,

> because seignorage revenues are such a small fraction of total government revenues. ... [This] neglects a more important channel ... the effects of monetary policy upon the real value of outstanding government debt, through its effects on the price level and upon the real debt service required, ... insofar as monetary policy can affect real as well as nominal rates.

Similarly, "fiscal policy is thought to be unimportant for inflation [because] inflation is a purely monetary phenomenon," or else because "insofar as consumers have rational expectations, fiscal policy should have no effect on aggregate demand." But this is not correct, Woodford argues: Even if people are individually rational, the economy as a whole can be "non-Ricardian" in the sense that changes in government spending will not be offset one for one by changes in private spending. "This happens essentially through the effects of fiscal disturbances upon private sector budget constraints and hence on aggregate demand." For this reason, "A central bank charged with maintaining price stability cannot be indifferent as to how fiscal policy is set."

debt ratio was stable but output was above the level consistent with price stability (in the eye of policymakers), so the application of the sound finance rule implied a large upward movement in interest rates.[9] Higher interest rates brought output to its desired level, but increased government interest payments, moving the economy off the debt-stability locus to a path of rising debt. Fiscal policy eventually responded to this monetary policy-induced rise in federal borrowing as the sound finance rule requires, by shifting the primary balance toward surplus. Large surpluses reduced aggregate demand, as became evident in the early 2000s, when interest rates were reduced to then-unprecedented levels in order to bring output up to potential. Low interest rates opened up space for the move toward primary deficits under Bush, which might have carried the cycle back toward its starting point if it had not been cut short by the collision of the interest rate instrument with the zero lower bound.

Table 1: Annual Contributions to Changes in the Federal Debt Ratio, Selected Periods

Period	Change in Debt-GDP Ratio	Due to...		
		Primary Balance	Interest Payments	GDP Growth
1950-1981	-1.6	-0.3	1.5	-3.0
1982-1989	1.8	1.0	3.4	-2.5
1990-2014	1.4	1.2	2.2	-2.0

Source: Kogan et al. (2015), authors' analysis.

The first column shows the average annual change in the federal debt-GDP ratio during the given period. The next three columns show the contributions of the primary balance, interest payments, and the growth of GDP respectively to the change in the debt ratio. The three columns do not sum exactly to the change in the ratio due to interaction effects. All values are in percentage points. Interest and growth rates are nominal; adding an inflation term would change the contributions of inflation and growth but would not affect the total debt change or the contribution of the primary balance.

In this context, it is important to realize that the majority of the rise in federal debt during the 1980s was due to higher interest rates, not to the tax and expenditure decisions of the Reagan administration. As Table 1 shows, over fiscal years 1950 through 1981 (the last pre-Reagan budget), the primary budget balance was on average in surplus of 0.3 percent of GDP, while interest payments averaged 1.5 percent of GDP, giving an average overall budget deficit of 1.2 percent of GDP. These deficits were more than offset by nominal growth of GDP, resulting in a decline of the debt-GDP ratio during this period of 1.6 percentage points per year.

[9] This is intended as an alternative way of describing, rather than an alternative explanation for, the Volcker shock.

Between fiscal 1982 and 1990, the overall federal deficit averaged 4.4 percent of GDP, with the average primary deficit equal to 1.0 percent of GDP and interest payments equal to 3.4 percent. During this period, the debt-GDP ratio rose by 1.8 points per year. In other words, while the annual change in the debt-GDP ratio was 3.4 points higher under the Reagan administration than in the preceding three decades, only about a third of this difference is attributable to increased spending and lower taxes. The majority of the difference is accounted for by higher interest payments.[10] (By contrast, the increase in the debt ratio over 2008-2014, not shown, is mainly attributable to a shift toward primary deficits.) The numbers reported in Table 1 are important for our analysis because they demonstrate that the cross-effects of changes in the policy interest rate on the federal debt ratio are quantitatively important.

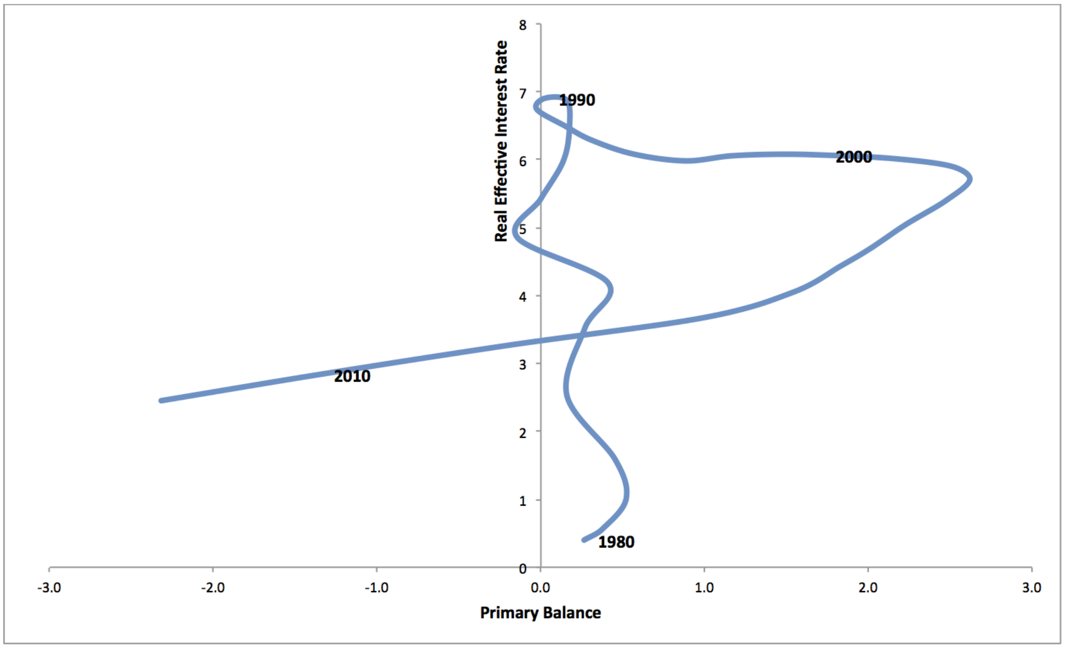
These movements are illustrated in Figure 7, which shows 5-year moving averages of the inflation-adjusted policy rate and the primary balance from 1971to 2013. The figure shows a clear counterclockwise movement, as predicted for policy interactions under a sound finance rule. Note again that the movement in the 1980s is mainly vertical.

Conclusions

The starting point of this paper is a simple observation: both the output gap and the trajectory of public debt-output ratio are jointly determined by both the fiscal balance and the interest rate set by the monetary authority. So both targets and both instruments must be analyzed within a single framework – rather than, as is more often the case, discussing the stabilization of output through monetary policy and the stabilization of public debt ratios through appropriate budget rules as if they were two independent questions. It follows that there is no such thing as a Wicksellian natural interest rate, but at best a schedule of such rates, one for each value of the primary balance. And similarly, we cannot specify a budget rule consistent with a stable debt-GDP ratio unless we also describe the behavior of (policy-determined) interest rates. It is no secret that periods of very high debt ratios have seen a shift in the primary target of monetary policy from price stability to the public debt ratio. (Reinhart, Kirkegaard and Sbrancia, 2011) But this historical fact is not well reflected in most formal discussions of macroeconomic policy.

[10]The data for these calculation is taken from the online appendix of Kogan et al. (2015). A similar argument is made there that most of the historical variation in the federal debt-GDP ratio is explained by changes in the interest rate on federal debt and in nominal GDP growth rates, and that the relative importance of these latter factors is greatest when the debt-GDP ratio is already high.

Figure 7: Price Stability and Debt Sustainability Conditions



The figure shows rolling 10-year averages for the primary balance and the inflation-adjusted effective interest rate on public debt. The labels show the ending date, so the starting point, at the bottom, represents average values for 1971-1980 while the ending point, on the left, shows the average values for the period 2005-2014. The trajectory is approximately a clockwise spiral, similar to what we suggest might be expected given destabilizing feedback between policy instruments under a "sound finance" policy rule.

From the perspective adopted here, the distinction between an orthodox "sound finance" instrument assignment and the alternative "functional finance" assignment takes on a different appearance. The case for functional finance does not depend on arguments about the economic costs, or lack thereof, of changes in the debt-GDP ratio, since that ratio can in general be held constant under either rule. If both policy instruments can be set instantly to their optimal values, then the two rules are in general equivalent. If the instruments are adjusted incrementally in response to deviations of the targets from their desired values, then the rules are distinguished by the different adjustment paths they follow. We show that while both policy rules converge at low debt-GDP ratios, only the functional finance rule converges at high debt ratios. Thus, counterintuitively, the case for countercyclical fiscal policy becomes stronger, not weaker, when public debt ratios are already high.

In the final part of the paper, we apply this framework to historical data for the postwar United States. We ask whether medium-term fluctuations can be explained, at least in part, by interactions between the two policy instruments. We tentatively suggest that the macroeconomic history of the past 40 years can be understood in these terms. Monetary tightening in response to inflation causes the debt ratio to increase, inducing (with a lag) a shift toward primary surpluses. The contractionary effects of surpluses lead the monetary authority to lower interest rates, which reduces debt service costs for the government, contributing to the fall in the debt ratio. Falling debt ratios encourage an increase in public spending, boosting demand until the monetary authority tightens again. And so on, at least potentially – only one full cycle is visible in the record. This is the result of each instrument being adjusted only in response to one of the targets, even though both instruments affect both; the result is cycles in policy space, with destabilizing economic effects. This suggests a greater role for endogenous policy cycles in macroeconomic fluctuations, and correspondingly lesser role for exogenous shocks, than in most accounts.

The framework offered here is limited in some important respects. Most obviously, we take no view on why, or whether, a stable debt-GDP ratio should be a target of policy. We simply accept this goal as a premise, since it is today a presupposition of most discussions of macroeconomic policy, and evidently shapes the choices of policymakers. We also ignore the effects of changes in output and inflation on the debt-GDP ratio, though these have been important historically. Taking these effects into account would probably strengthen the case for endogenous interactions between policy instruments as a source of macroeconomic instability, since it introduces another channel by which an increase in the policy interest rate can raise the debt-output ratio. Perhaps most importantly, we ignore open-economy complications. For the US, this is probably not a serious limitation. But for most other countries it is unclear whether the analysis here would be meaningful, at least as applied to concrete historical developments, without considering the balance of payments and exchange rate fluctuations. In small open economies, the exchange rate is, at least potentially, a target for policy on a level with the output gap and the debt ratio. (Ghosh, Ostry and Chamon, 2015) Clearly this dimension cannot be ignored when a significant fraction of public debt is financed in a foreign currency. In a somewhat different direction, this paper invites, but does not attempt to answer, a political economy question: If the sound finance and functional finance rules are formally equivalent, why is there such a strong commitment – among both policymakers and the economics profession – to the idea that the fiscal balance instrument must be assigned to the debt ratio and the interest rate instrument to the output gap? Answering this question would be an important step in understanding the political constraints on macroeconomic policymaking, which may in the end be more important than the economic constraints explored here.

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